QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 141 **Spring 2023**

Instructions: Answer all questions. Show all work.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

   a) $\lim\limits_{x \to 2} \dfrac{4 - x^2}{2x^2 - 5x + 2}$

   b) $\lim\limits_{x \to -5} \dfrac{\sqrt{2x + 19} - 3}{x + 5}$

   c) $\lim\limits_{x \to -3^+} \dfrac{x^2 - 4x - 21}{x^2 + 6x + 9}$

   d) $\lim\limits_{x \to 6} \dfrac{\frac{3x}{x-2} - \frac{18}{4}}{x - 6}$

   e) $\lim\limits_{x \to -\infty} \dfrac{5x^4 + 8x^5 - 4}{2x^4 - 3x - 11x^5}$

2. Using the TABLE feature on your calculator, either estimate the following limit or determine that it is $+\infty$, $-\infty$, or does not exist. Include at least 8 lines in your table to justify your answer.

$$\lim_{x \to 4} \frac{-x}{2x - 8}$$

3. a) Show that the equation $3x^5 + 2x^3 + 7x = 25$ has exactly one real root. Name any theorems or properties used in your explanation.
   b) Use your graphing calculator to find the root in part a), accurate to <u>three</u> decimal places.

4. a) <u>Using only the definition of derivative</u>, find $f'(x)$ if $f(x) = \sqrt{3x - 5}$.
   b) Find an equation of the tangent line to the curve $f(x)$ at the point where $x = 2$.

5. In each of the following, find $\frac{dy}{dx}$. (Algebraic simplification is not needed.)

   a) $y = 6x^5 - 15\sqrt[3]{x} + \dfrac{4}{x^2} - \cos x + 5\pi^3 - 8x + 9$

   b) $y = (7x^4 - 3x^7)^6 (5x - 2)^3$

   c) $y = \dfrac{\tan(2x^5)}{3x - 1}$

   d) $y = \cos(\sin(4x^3))$

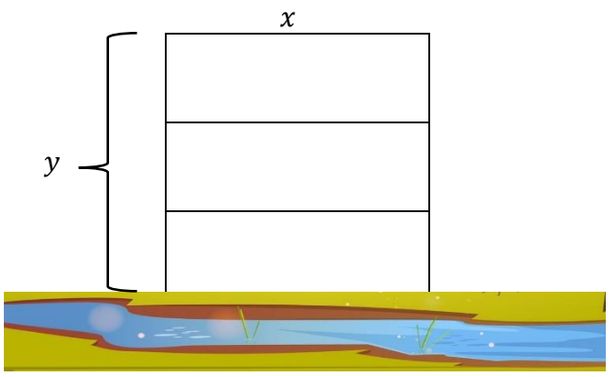
   e) $y = 6x^9 \csc x + \cot^5 x$

   f) $4x^3 \sin y = x^5 y^3 - 7x$

(continued on the back)

6. A triangle has a base of 20 cm and a height of 16 cm. The base begins to increase at the rate of 4 cm/sec and at the same instant the height begins to decrease at the rate of 2 cm/sec. How fast is the area of the triangle changing 3 seconds later? Is the area increasing or decreasing?

7. A farmer has 120 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. He also wants to divide the field into three pens by placing two fences parallel to the river. What are the dimensions of the field that has the largest area? What is the maximum area of this field?



8. Let $f(x) = 12x^3 - 3x^4 - 10$. Using calculus and showing all necessary work,
   a) find the intervals of increase and intervals of decrease of f.
   b) find the local maximum and minimum values of f, if any.
   c) find the intervals where f is concave up and those where f is concave down.
   d) find any and all inflection points of f.
   e) Use the information found in parts a) through d) to sketch the graph of $y = f(x)$.