

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Math 142

Spring 2023

Instructions: Answer each question and show your work.

1. Use a Riemann sum with 6 subintervals and right endpoints to approximate the integral

$$\int_0^3 (x^3 - 6x) dx.$$

2. Find the derivative $\frac{dy}{dx}$ for each of the following functions. (Algebraic simplification is unnecessary.)

a) $y = \sqrt{6 + 2e^{4x}}$

b) $y = \int_{-x}^x \left(\frac{1}{t^3 + 1} \right) dt$

c) $y = \sin^{-1}(\cos \sqrt{x})$

d) $y = \frac{(x^2 + 3)^4}{(5x^5 - 2)^2(3x^2 - 5)^5}$ [Hint: Logarithmic Differentiation]

3. a) Find each of the following indefinite integrals:

i) $\int \frac{6x^5 - \sqrt{x} + 5x^2}{x^3} dx$

ii) $\int (6x - 3)(4x^2 + 8) dx$

iii) $\int \frac{\sec^2\left(\frac{1}{x^3}\right)}{x^4} dx$

- b) Without using your calculator, find the exact value of each of the following definite integrals:

i) $\int_0^1 \frac{e^{3x}}{1 + e^{6x}} dx$

ii) $\int_1^5 |x - 2| dx$

4. Let R be the region in the plane bounded by the curves $y = 3x$ and $y = 3\sqrt{x}$.

a) Find the area of R .

b) Find the volume of the solid generated by rotating R about

i) the y -axis

ii) the line $y = 3$

(continued on the back)

5. a) Use the definition of the integral as the limit of Riemann sum to evaluate

$$\int_{-2}^0 (4x^2 + 4x) dx.$$

$$\left(\text{Note: } \sum_{i=1}^n i = \frac{n(n+1)}{2} , \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

- b) Find the average value of $f(x) = 4x^2 + 4x$ on the interval $[1,3]$.

6. The half-life of cesium-137 is 30 years. Suppose we have a 70-mg sample.
- Find the mass that remains after t years.
 - How much of the sample will remain after 130 years? (Round your answer to two decimal places.)
 - After how long will only 1 mg remain? (Round your answer to one decimal place.)
7. Find the exact length of the curve $x = \frac{1}{3}\sqrt{y}(y-3)$, where $1 \leq y \leq 25$.
8. Solve the differential equation $y' = (2x+3)(y^2+4)$ if $y(0) = -1$.