QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 | Spring 2023

Instructions: Answer all questions. Show all work.

1. Compute the following integrals:

   (a) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

   (b) $\displaystyle\int \sin^3 x \cos^4 x\,dx$

   (c) $\displaystyle\int_1^3 xe^{2x}\,dx$

   (d) $\displaystyle\int \frac{x+1}{x^2(x^2+1)}\,dx$

2. Determine if the following improper integral converges, or diverges:

$$\int_{-4}^{0} \frac{1}{\sqrt{x+4}}\,dx.$$

   If it converges, find its exact value.

3. Compute the following limits:

   (a) $\displaystyle\lim_{x\to\infty} x^2 e^{-x^3}$

   (b) $\displaystyle\lim_{x\to 0^+} (1+x)^{\cot x}$

4. Determine whether each series is absolutely convergent, conditionally convergent, or divergent:

   (a) $\displaystyle\sum_{n=1}^{\infty} 2(3^{-n})$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2n+4}{n+7}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{2n+1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{7n}{n^{5/2}}$

5. Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{4^n n} x^n.$$

(continued on the back)

6. Starting with the Maclaurin series for $\dfrac{1}{1-x}$, find the Maclaurin series for the function

$$f(x) = 1/(1 - 125x^3).$$

What is its radius of convergence?

7. Use term-by-term integration of power series to obtain a numerical approximation to $\displaystyle\int_0^1 e^{-x^3}\,dx$ with an error of less than $.0001 = 10^{-4}$. Justify your answer.

8. Let $f(x) = \sqrt[4]{x}$, for $x \geq 0$.

   (a) Write the third Taylor polynomial, $T_3(x)$, for $f(x)$ centered at $a = 16$.

   (b) Using Taylor's Formula, write the expression for the general remainder $R_3(x) = f(x) - T_3(x)$, for any x in the interval $[12, 20]$, and some number z in this interval.

   (c) Determine if the approximation $\sqrt[4]{x} \approx T_3(x)$ has error less than 10^{-3}, for all x in the interval $[12, 20]$. To do so, give an explicit numerical upper bound for $|R_3(x)|$ for such values of x.