QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 151 **Spring 2023**

Instructions: Answer all questions. Show all work.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

   a) $\lim_{x \to 9} \dfrac{9 - x}{x^2 - 7x - 18}$

   b) $\lim_{x \to -3} \dfrac{\sqrt{x + 12} - 3}{x + 3}$

   c) $\lim_{t \to 0} \dfrac{\tan(10t)}{\sin(5t)}$

   d) $\lim_{x \to \infty} \dfrac{4x + 2}{\sqrt{25x^2 + 1}}$

   e) $\lim_{x \to 5} \dfrac{x^2 - 2x - 15}{|x - 5|}$

2. Let $f(x) = \dfrac{4x}{x + 8}$. Using the definition of the derivative, find $f'(x)$.

3. In each of the following find $\frac{dy}{dx}$. (You need not simplify.)

   a) $y = \sqrt[3]{5x^2 + \tan x}\,(3x^4 + 3x + 6)^4$

   b) $y = \dfrac{\tan 5x}{1 + \sec 2x}$

   c) $y = \cos\left(\tan\left(\sqrt{\sin 4x}\right)\right)$

   d) $\cos(x + y) = y^4 \sin x$

   e) $y = \displaystyle\int_{x^3}^{9} \sqrt{16 + t^2}\, dt$

4. a) Show that the equation $x^5 + 2x - 1 = 0$ has exactly one real root. Justify your conclusion by using appropriate theorems.

   b) Use a graphing calculator, estimate the real zero of $y = x^5 + 2x - 1$ to three decimal places.

5. At 9:00 am, car A is 160 miles north of car B. Car A is traveling south at 45 miles per hour and car B is traveling east at 20 miles per hour. How fast is the distance between the cars changing at 12:00 pm?

(continued on the back)

6. Let $f(x) = \dfrac{2x}{x^2 - 4}$.

a) For which intervals is f increasing and for which is f decreasing?
b) Find all local maxima and/or local minima of f.
c) Find any and all vertical and horizontal asymptotes of the graph of f.
d) For which intervals is the graph of f concave up and for which is it concave down?
e) Find the inflection point(s) of the graph of f, if any.
f) Sketch the graph of $y = f(x)$ using the information found in parts a) – e).

7. A box is to be constructed with a square base and no top. Material for the sides costs \$ 2 per square foot and material for the base costs \$ 8 per square foot. If the construction cost of the box is \$ 2,400, find the dimensions of the box which yield the greatest volume.

8. Find each of the following integrals.

a) $\displaystyle\int \tan^5 x \sec^2 x \; dx$

b) $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

c) $\displaystyle\int \frac{x}{\sqrt{2-x}}\, dx$

9. Evaluate $\displaystyle\int_0^3 (x^2 + 2x)dx$ as the limit of a Riemann sum.

(Note: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$)