QUEENS COLLEGE

Department of Mathematics
Math 152 Final Exam, Spring 2023

- **The exam has two parts.**
- **You have 150 minutes to answer the questions.**
- **Show all work in your exam book.**

Part I [30 points]. Write the letter of the correct answer in your exam book. Show your work.

1. If $f(x) = e^{5x} + x$ and f^{-1} denotes the inverse of f, the derivative $(f^{-1})'(1)$ is

(A) $5e^5 + 1$ (B) $\dfrac{1}{5e^5+1}$ (C) 6 (D) $\dfrac{1}{6}$

2. A cup of boiling water is left in a 70°F room. Which of the following formulas could describe the temperature T of the water after t hours?

(A) $T = 212\,e^{-4t} + 70$ (B) $T = 142\,e^{-4t} + 70$

(C) $T = 70\,e^{-4t} + 212$ (D) $T = 70\,e^{-4t} + 142$

3. $\displaystyle\lim_{x\to 0} \frac{\tan^{-1}(3x) - 3x}{x^3}$ is

(A) -1 (B) -3 (C) -9 (D) $-\infty$

4. I set up an integral for the arc length of $y = \cos x$ between $x = 0$ and $x = \pi/4$. My calculator estimates this integral to be

(A) 0.8793 (B) 0.8671 (C) 0.8520 (D) 0.8443

5. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n}(x-1)^n$ is

(A) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (B) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (C) $\left[\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{3}{2}\right]$

6. Ignoring the constant of integration, the power series of $\displaystyle\int \frac{1}{1+x^5}\,dx$ centered at 0 has the form

(A) $x - \dfrac{x^6}{6} + \dfrac{x^{11}}{11} - \dfrac{x^{16}}{16} + \cdots$ (B) $x + \dfrac{x^6}{6} - \dfrac{x^{11}}{11} + \dfrac{x^{16}}{16} - \cdots$

(C) $x - \dfrac{x^5}{5} + \dfrac{x^{10}}{10} - \dfrac{x^{15}}{15} + \cdots$ (D) $x + \dfrac{x^5}{5} - \dfrac{x^{10}}{10} + \dfrac{x^{15}}{15} - \cdots$

(continued on the other side)

Part II [70 points]. Solve the following six problems.

Problem 1. [12 points] In each case, find the derivative $y' = \dfrac{dy}{dx}$:

(a) $y = \ln(x + \tan^{-1} x)$

(b) $y = \sin^{-1}(e^{2x})$

(c) $y = (1 + x)^{\cos x}$ (Hint: Use logarithmic differentiation)

Problem 2. [15 points] Evaluate the following integrals:

(a) $\displaystyle\int \sqrt{x} \ln x \, dx$

(b) $\displaystyle\int \frac{2x + 3}{x^2 - 9} \, dx$

(c) $\displaystyle\int_1^{\infty} x^2 e^{-x^3} \, dx$

Problem 3. [10 points] Let R be the region in the plane bounded by the curve $y = e^{-x}$ and the lines $y = -x + 1$ and $x = 1$.

(a) Sketch R and find its area.

(b) Find the volume of the solid obtained by rotating R around the x-axis.

Problem 4. [8 points] Find the explicit form of the solution of the differential equation

$$\frac{dy}{dx} = (1 + \sec^2 x)\, y^{2/3}$$

which satisfies the condition $y(\pi) = 0$.

Problem 5. [15 points] Using appropriate tests, determine the convergence or divergence of each of the following series:

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^n \, n!}$

Problem 6. [10 points]

(a) Using your knowledge of the Maclaurin series of the cosine function, find the Maclaurin series of $f(x) = x\cos(x^3)$.

(b) Use your answer to (a) to write the definite integral

$$I = \int_0^1 x\cos(x^3)\, dx$$

as an alternating series. Then estimate the value of I with an error of less than 0.0001.