

**Queens College
Department of Mathematics**

**Final Examination
2.5 Hours**

Mathematics 110

Fall 2023

Instructions: Answer all questions. Show all work in the exam booklet. Good luck!!

1. (a) Suppose an election is being held among five candidates: Luffy, Nami, Zoro, Usopp, and Sanji. After the first 76 votes are counted, the tallies are as follows:
- Luffy 20
Nami 16
Zoro 14
Usopp 12
Sanji 14
- If 24 votes remain, what is the minimal number of remaining votes Nami can receive and be assured of a win?
- (b) If 504 votes are cast in an election, what is the smallest number of votes a winning candidate can have in a five-candidate race to be decided by plurality?
2. 200 people are voting for which fictional character is the most powerful. Their preference rankings are listed below. Approval is indicated with \checkmark :

Candidate	Number of Votes						
	40	20	20	20	30	40	30
Featherine A. Aurora	1 \checkmark	1 \checkmark	1 \checkmark	3	3	4	4
Superman	4	4	2 \checkmark	1 \checkmark	1 \checkmark	3	3
Thanos	3	2 \checkmark	3 \checkmark	4	2 \checkmark	1 \checkmark	2 \checkmark
Enrico Pucci	2 \checkmark	3 \checkmark	4	2 \checkmark	4	2 \checkmark	1 \checkmark

- (a) Which candidate would be selected using the plurality method?
- (b) Which candidate would be selected using the plurality method with a runoff between the top two finishers?
- (c) Which candidate would be selected using Borda's Method?
- (d) Which candidate would be selected using an approval vote?
3. In a country with four states, the population of each state is listed below. There are 25 seats for representatives to be allocated to the states based on their populations.

State	Gridania	Limsa	Ul'dah	Ishgard
Population	2500	3400	3000	2900

- (a) Apportion the representatives using Hamilton's Method.
- (b) Apportion the representatives using Lowndes' Method.
- (c) Apportion the representatives using Webster's Method.
- (d) How many representatives would a state with a natural quota of 9.4935 be initially allocated using Hill-Huntington's method?

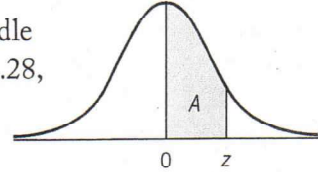
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4. Find the sample mean, and the sample standard deviation of the numbers: 23, 17, 50, 34, 26.
5. A random sample of $n = 24$ grades on a history exam is listed below.
- | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 70 | 55 | 77 | 83 | 80 | 68 | 98 | 72 | 87 | 91 | 66 | 82 |
| 75 | 58 | 69 | 78 | 77 | 77 | 75 | 89 | 86 | 71 | 95 | 81 |
- (a) Identify the five number summary of this data.
(b) Identify the mode of this data.
(c) What is the range of this data?
(d) Construct a histogram with the first class interval 50-59 and identify the modal class.
6. Suppose a game is being played where a fair, six-sided die marked with numbers 1-2-3-4-5-6 is rolled. Each number has an equal chance of being an outcome. In this game, you place a bet on one number. It costs \$3 to place a bet on one number, and you win \$9 if the die lands on your number:
- (a) Construct the probability distribution of this game.
(b) Find this game's expected value in dollars.
7. A password can be made from capital letters (A - Z) and digits (0 - 9). How many 6-character long passwords are possible if:
- (a) repetition of letters and digits is allowed?
(b) repetition of letters and digits is allowed, but the password must be three letters followed by three digits?
(c) only letters may be used, and no repetition is allowed?
8. A bag contains 20 marbles. Five marbles are green and numbered 1 - 5, five marbles are blue and numbered 1 - 5, and 10 marbles are red and numbered 1 - 10. Find the probability that if:
- (a) One marble is randomly drawn from the bag, it is blue, OR marked with #1.
(b) One marble is randomly drawn from the bag, it is blue, AND marked with #1.
(c) Three marbles are randomly drawn from the bag with replacement, the first two are red, and the third is blue.
(d) Three marbles are randomly drawn from the bag without replacement, the first two are red, and the third is blue.
9. Scores on a Bird Law exam have an approximately normal distribution with a mean of 70 points and a standard deviation of 5.
- (a) Find the percent of scores below 78.
(b) Find the percent of scores above 70.
(c) If 1000 students took the exam, about how many students scored between 63 and 65?
(d) Below what grade can you find 90.32% of all grades?

Statistical Tables

Table A Normal Curve (z) Table

The normal curve table gives only the percentage of data starting from the middle ($z = 0$), out to whatever z score you look up. For instance, if you look up $z = 1.28$, you get .3997. This means .3997 or 39.97% of the data in the normal curve is found between $z = 0$ and $z = 1.28$.



Normal										
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998