

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
Final Examination
2 ½ hours

Math 115

Fall 2023

Instructions: Show algebraic solutions and simplifications.

1. Consider two points in the coordinate plane $A(6, 7)$ and $B(-1, 21)$.
 - a. Write an equation of the line that goes through points A and B.
 - b. Write an equation of the vertical line that goes through point A.
 - c. Write an equation of the horizontal line that goes through point B.
 - d. Find the coordinates of the midpoint of the line segment \overline{AB} .
 - e. Find the length of the line segment \overline{AB} .

2. Line C has equation $3x - 4y = 12$
 - a. Find the coordinates of its x -intercept.
 - b. Find the coordinates of its y -intercept.
 - c. Use the x - and y -intercepts to sketch the graph of Line C.
 - d. Find the slope of a line parallel to line C.
 - e. Find the slope of a line perpendicular to line C.

3. Write an equation of the circle centered at $(0, 0)$ with radius length = 1.

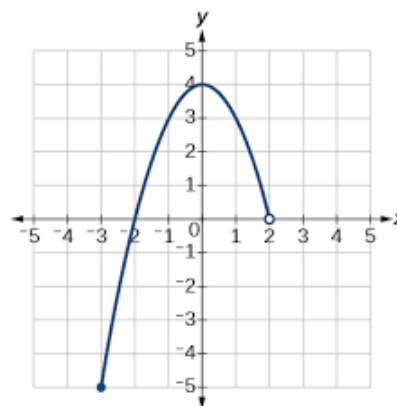
4. Given that $f(x) = 2x^2 - 4x + 2$,
 - a. find and simplify $f(-1)$
 - b. find and simplify $f(a + h)$
 - c. write an equation for the axis of symmetry of the graph of $f(x)$.
 - d. is the vertex a minimum or maximum point?
 - e. what are the coordinates of the vertex?

5. Given that $g(x) = \sqrt{x - 5}$,
 - a. Evaluate $g(x)$ at $x = 9$
 - b. Write the domain of $g(x)$ in interval notation.
 - c. Write a number **not** in the domain of $g(x)$.

6. Given that $h(x) = \frac{2}{x-4}$,
 - a. Find $h(6)$
 - b. Write the domain of $h(x)$ in interval notation.

7. Use the graph of $y = f(x)$ shown to the right.
 - a. Find $f(0)$.
 - b. Find the domain of $f(x)$ in interval notation.
 - c. Find the range of $f(x)$ in interval notation.
 - d. Find the value of x for which $f(x) = 0$.
 - e. Solve $f(x) = 3$.
 - f. Write an equation of the axis of symmetry.

8. Factor completely. Otherwise, write PRIME POLYNOMIAL.
 - a. $x^2 + 9$
 - b. $16x^2 - 9$
 - c. $x^2 - 4x + 5$
 - d. $6x^3y - 15x^2y + 6xy$



(continued on the back)

9. Find all real solutions, in simplest form, for the equations below.
- $x^2 - 2x = 15$
 - $x^3 - x^2 - x + 1 = 0$
 - $6x^2 + 4x - 3 = 0$
 - $\sqrt{2x + 7} + 2 = x + 4$
10. Solve for x : $\frac{2x - 3}{3x - 4} = y$
11. Simplify: $\frac{4x^3 + 6x^2 - 2x}{2x}$
12. Divide: $\frac{2x^2 + x - 3}{6x^2 - 2x} \div \frac{2 - 2x}{6x - 2}$
13. Use long division to divide $(8x^3 + 27) \div (2x + 3)$
14. Simplify: $\frac{1 + \frac{2}{x}}{1 - \frac{2}{x}}$
15. Combine and simplify:
- $-\frac{1}{x + 2} + 1$
 - $\frac{3}{x - 2} - \frac{5}{x + 2} - \frac{6x}{x^2 - 4}$
16. Simplify $(\sqrt{2x - 3})^2 - (\sqrt{2x} - \sqrt{3})^2$
17. Write the following as equivalent expressions with positive exponents:
- $2x^{-3}$
 - $-2x^{-3}$
 - $(2x)^{-3}$
 - $\frac{2}{x^{-3}}$
18. Simplify and write your answer with positive exponents only:
- $$\frac{(3xy)^{-3}(4x^8y^2)^{\frac{1}{2}}}{(x^{12}y^{-12})^{-\frac{1}{3}}}$$
19. Rationalize each denominator and simplify
- $$\frac{21}{3 - \sqrt{2}} - \frac{6}{\sqrt{2}}$$
20. Simplify: $6x^2y\sqrt{80x^2} + \sqrt[3]{27x^3y^6} + 3y\sqrt{45x^6}$
21. A student spent 30 minutes on the rowing machine in the gym and 20 minutes lifting weights—he burned 430 calories. During his next visit to the gym, he spent 50 minutes on the rowing machine and 10 minutes lifting weights—he burned 600 calories.
- Let R represent the number of calories burned per minute on the rowing machine. Let W represent the number of calories burned per minute lifting weights. Write a system of two linear equations to represent the student's exercise over two days.
 - Use algebra to solve the system of linear equations to determine how many calories he burned per minute on the rowing machine and how many calories he burned per minute lifting weights.