Final Examination
$2\frac{1}{2}$ Hours

Mathematics 122   **Fall 2023**

Instructions: Answer all questions. Show all work.

1. Let $f(x) = -2x^2 + 4x + 6$.
   a) Express $f(x)$ in standard form. (Vertex form.)
   b) Sketch the graph of $f(x)$. Clearly label the coordinates of the vertex and any x- and y-intercepts.
   c) State the maximum or minimum value for $f(x)$.

2. Let $f(x) = 2x^2 - 7x$. Find and simplify
   a) $f(a)$
   b) $f(a+h)$
   c) $\dfrac{f(a+h) - f(a)}{h}$ $(h \neq 0)$

3. Determine the domain for each of the following functions. Give your answer in interval notation.
   a) $g(x) = \dfrac{x+4}{3x^2 - 21x}$
   b) $h(x) = \dfrac{5-x}{\sqrt{3x-9}}$
   c) $t(x) = 2\log(x+3)$

4. Given the polynomial function $P(x) = -2x^3 + 12x^2 - 18x$.
   a) Find all zeros of P and determine the multiplicity of each zero.
   b) Sketch the graph of $y = P(x)$. Include the zeros, and label the x- and y-intercepts.
   c) What is the end behavior of $P(x)$?

5. Given the rational function $R(x) = \dfrac{3}{x-4} + 3$.
   a) Sketch the graph of $R(x)$.
   b) Include any asymptotes and write equations of its vertical and horizontal asymptotes.
   c) Label the x- and y-intercepts.
   d) Find $R(5)$ and $R(10)$ and include these points on the graph.

6. Let $f(x) = \frac{1}{3}x^2 - 2$, with domain $x \geq 0$.
   a) Sketch the graph of $f(x)$
      i) include the x- and y-intercepts. (Round to one decimal place, if necessary.)
      ii) Find $f(3)$ and $f(6)$ and include these points in the graph.
   b) What is the range of $f(x)$?
   c) On the same set of axes, graph and label the line $y = x$.
   d) On the same set of axes, graph $f^{-1}(x)$.
   e) Find $f^{-1}(x)$ algebraically.
   f) What are the domain and range of $f^{-1}(x)$?

(continued on the back)

7. Let $f(x) = 12 - 5x^2$ and $g(x) = 2x - 3$.
   a) Evaluate $(g \circ f)(-2)$.
   b) Find $(f \circ g)(x)$ and <u>simplify</u>.
   c) Find $g^{-1}(x)$ and show algebraically that $g^{-1}(x)$ is the inverse of $g(x)$.

8. Solve each of the following for x.
   a) $4^{x+8} = 32^{x-1}$

   b) $3e^{7x} = 14$ (Round your answer to 5 decimal places.)

   c) $\log_3(2x - 5) + \log_3 x = 1$

   d) $2\cos^2 x = \sin x + 1$ on the interval $[0, 2\pi)$

9. Find the exact value for each expression, without using your calculator.
   a) $\log_2 12 + 2\log_2 2 - \log_2 3$

   b) $\tan\left(\cos^{-1}\left(\dfrac{5}{13}\right)\right)$

   c) $\sin\left(\dfrac{5\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right) - \cos\left(\dfrac{5\pi}{12}\right)\sin\left(\dfrac{\pi}{12}\right)$

10. Given $\sin A = \dfrac{7}{25}$ and $\angle A$ is in Quadrant II and $\cos B = -\dfrac{4}{5}$ and $\angle B$ is in Quadrant III, find
    a) $\cos(A + B)$
    b) $\sin(2A)$
    c) $\cot B$
    d) $\cos^2 B$

11. Verify the identity
    $$\dfrac{\sin x}{1 + \cos x} + \cot x = \csc x$$

12. Sketch the graph of $y = -3\cos\left(\dfrac{1}{2}x\right)$ on the interval $[0, 2\pi]$. Label its x-intercept and the y-intercept.