

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 131

Fall 2023

Instructions: Answer all questions and show all your work.

1. Using algebraic methods (not the calculator), find each of the following limits. If a limit is ∞ , $-\infty$, or does not exist, state this as your answer.

a)
$$\lim_{x \rightarrow 2} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$$

b)
$$\lim_{x \rightarrow -\infty} \frac{1 - 3x^3}{2x^3 - 6x + 2}$$

c)
$$\lim_{x \rightarrow 0^-} \frac{x+1}{x}$$

d)
$$\lim_{x \rightarrow 4} \frac{16 - x^2}{x^2 + x - 20}$$

2. a) Define what it means for a function $f(x)$ to be continuous at $x = a$.
- b) Let $f(x) = \begin{cases} 1 - x, & x < 1 \\ x - x^2, & x \geq 1 \end{cases}$. Is the function continuous on $(-\infty, \infty)$? Explain why or why not.

3. Using the definition of derivative, find $f'(x)$ if $f(x) = \sqrt{2x}$.

4. Find the derivative of each of the following functions. (You do not need to simplify.)

a)
$$y = 1 + \ln(x^3 + 1) - \sqrt[5]{x^3} + \frac{2}{\sqrt{x}} + 3x^2$$

b)
$$y = \frac{e^{-5x^2}(2x-1)^3}{(x-5)^2}$$
 (use logarithmic differentiation)

c)
$$y = \sqrt{\frac{3x^3 + 2x - 4}{1 - x^2}}$$

d)
$$xe^{-y} + ye^{-x} = 3$$

5. Use the Intermediate Value Theorem to show that $f(x) = x^5 - x^2 - 4$ has at least one root on the interval $[1, 2]$.

6. A tumor is modeled as being roughly spherical, with radius r . If the radius of the tumor is currently $r = 0.54$ cm and is increasing at the rate of 0.13 cm per month, what is the corresponding rate of change of its volume? (Hint: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.) Round your answer to 2 decimal places.

(continued on the back)

7. For a small farm, the cost (in dollars) to produce x containers of ice cream is $C(x) = 0.3x + 37$. The demand function of ice cream is $p(x) = 8.5 - 0.005x$, where p is the unit price of the ice cream.
- Find the profit function.
 - How many containers of ice cream need to be sold to maximize the profit? What is the maximum profit?
8. A manufacturer estimates that if x units of a particular commodity are produced, the total cost will be $C(x)$ dollars, where $C(x) = x^3 - 24x^2 + 350x + 338$.
- What is the actual cost of the 25th unit?
 - Use marginal analysis to estimate the cost of the 25th unit.
9. Given $f(x) = x(2x - 3)^2$.
- On what intervals is f increasing? decreasing?
 - On what intervals is the graph of f concave up? concave down?
 - Find the coordinates of the relative extrema and inflection point(s) of f .
 - Sketch the graph of $f(x)$.
10. A bank pays 5% interest compounded quarterly, and a savings institution pays 4.9% interest compounded continuously.
- If you invested \$3000 in the bank and the same amount in the savings institution, which account will be worth more in 5 years? How much more? (Round your answer to the nearest cent.)
 - If you decide to invest in the savings institution, how long will it take to double the amount of money you invest? (Round your answer to two decimal places.)