

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 141

Fall 2023

Instructions: Answer each question in the blue book. Show your work and justify your answers.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

a) $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{25 - x}$

b) $\lim_{x \rightarrow \infty} \frac{\sqrt{5 + 6x^4 + 2x^6}}{-x^3 + 2x - 1}$

c) $\lim_{x \rightarrow 7^-} \frac{x^2 - 49}{x^2 - 14x + 49}$

d) $\lim_{x \rightarrow 0} \frac{\tan(2x)}{3 \sin(5x)}$

2. Use a calculator to generate a table to evaluate $\lim_{x \rightarrow 0} \frac{\cos x + \sin x - 1}{5 \sin x}$.

3. Consider the following piecewise function.

$$f(x) = \begin{cases} x^2 - 2 & x < -1 \\ 3x & -1 \leq x < 1 \\ 2x^3 + 1 & x \geq 1 \end{cases}$$

- a) Is $f(x)$ continuous at $x = -1$? Justify your answer.
b) Is $f(x)$ continuous at $x = 1$? Justify your answer.
4. a) Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{x^2}$.
b) Using the result found in part (a), find an equation of the tangent line to the graph of $y = f(x)$ at the point $(-1, 1)$.

5. Find $y' = \frac{dy}{dx}$ for each of the following. (Algebraic simplification is not needed.)

a) $y = \sqrt[4]{x} - 8x^2 + \frac{1}{x^5} - \pi$

b) $y = 5x \sec(x^2)$

c) $y = \frac{\cos\left(\frac{1}{x}\right)}{\tan(\sqrt[3]{x})}$

d) $x^4 + \sqrt{xy^3} = y^2 + 2$

(continued on the back)

6. Two trains start their trips in London at 12 noon. Train A travels north at 24 km/hr while train B travels west at 70 km/hr. How fast is the distance between the trains increasing at 3:00 PM?
7. Use linear approximation or differentials to obtain an estimate for $\sqrt[4]{16.3}$.
8. Using appropriate theorems, show the function $f(x) = 3x + \cos x - 5$ has exactly one root.
9. A bin for holding heavy material must be in the shape of a rectangular box with a square base and open top. Material for the square base costs \$5 per square foot, while material for the four sides costs \$2 per square foot. If the volume of the bin must be 108 cubic feet, find the dimensions of the bin that will minimize the cost of constructing it.
10. Consider the graph of $f(x) = \frac{4x}{x^2 + 4}$. Using calculus and showing all necessary work,
- find the domain of $f(x)$.
 - find the intervals on which f is increasing and those on which f is decreasing.
 - find and classify all extrema of $f(x)$.
 - find the intervals on which $f(x)$ is concave upward and those on which it is concave downward.
 - find any inflection points on $f(x)$.
 - Use the information found in parts a) through e) to sketch the graph of $y = f(x)$.