

**QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS**

**FINAL EXAMINATION**

**$2\frac{1}{2}$  HOURS**

**MATHEMATICS 142**

**FALL 2023**

**INSTRUCTIONS: SHOW ALL THE WORK IN YOUR BLUE BOOK FOR ALL QUESTIONS.**

1. Find  $y' = \frac{dy}{dx}$  for each of the following. (Algebraic simplification is not needed.)
  - a)  $y = \ln \left[ \frac{\ln x}{(6x + 5)^3(x^3 - 2)} \right]$
  - b)  $y = \sin^{-1}(e^{3x})$
  - c)  $y = 10^{5x^2} + \log_3(4x^5)$
  - d)  $y = [\cos(4x)]^{6x}$
  - e)  $y = \int_2^{-x^3} (t^4 + 3 \sin^2 t) dt$
  
2. Let  $f(x) = x^5 + 7x^3 + 6x + 5$ .
  - a) Show that  $f(x)$  is one-to-one, hence has an inverse,  $f^{-1}(x)$ .
  - b) Find  $(f^{-1})'(19)$ .
  
3. Using the definition of the definite integral as the limit of Riemann sums, evaluate  $\int_0^2 (3x^2 + 4x - 1) dx$ .  
**Note:**  $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$  ;  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$  ;  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$
  
4. Find the following integrals:
  - a)  $\int \left( 6x^2 + 7x\sqrt{x} + \frac{3}{x} - 5 \right) dx$
  - b)  $\int \frac{(e^{2x} + 1)^2}{e^{2x}} dx$
  - c)  $\int x^3 \sec(x^4) \tan(x^4) dx$
  - d)  $\int \frac{x + 2}{x^2 + 4x - 9} dx$
  - e)  $\int \frac{e^{2x}}{1 + e^{4x}} dx$
  
5. Let  $R$  be the region in the plane bounded by the curves  $y = x^2 - 6x + 8$  and  $y = x + 2$ .
  - a) Find the area of the region  $R$ .
  - b) Find the volume of the solid of revolution obtained when  $R$  is rotated about the line  $y = -1$ .
  - c) Find the volume of the solid of revolution obtained when  $R$  is rotated about the  $y$ -axis.
  - d) Set up the integral for the perimeter of the region  $R$  and then use your calculator to approximate the result to three decimal places.
  
6. Solve the differential equation  $y' = e^{-y} (2x - 4)$  with initial conditions  $x = 5$  and  $y = 0$ .
  
7. The half-life of the radioactive isotope actinium-225 is 10 days. A scientist stores 2,000 mg of this isotope for later use.
  - a) Find a formula that computes the mass that remains after  $t$  days.
  - b) How much of the sample remains after 3 weeks? (Round your answer to the nearest hundredth.)
  - c) After how many days will the sample decay to 300mg? (Round your answer to the nearest day.)