QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 Fall 2023

Instructions: Show your work. Phones should be away.

1. Compute:
   (a) $\displaystyle\int (\ln x)^2 \, dx$
   (b) $\displaystyle\int \tan^4(2x) \, dx$
   (c) $\displaystyle\int \sqrt{36 - x^2} \, dx$
   (d) $\displaystyle\int \frac{x^3}{x^2 + 2} \, dx$
   (e) $\displaystyle\int \frac{3x^2 - 4x + 2}{x(x-1)^2} \, dx$

2. Find the exact value of the following limit:
$$\lim_{x \to 0^+} (2 - e^{3x})^{1/x}$$

3. Determine, without the use of a calculator, whether or not each of the following sequences converges or diverges. If a sequence converges, find what it converges to. If a sequence diverges, state that. Justify your answer in each case.
   (a) $\displaystyle\left\{ \frac{m(2m)!}{(2m+1)!} \right\}$
   (b) $\displaystyle\left\{ \frac{(-1)^n \sin 3n}{\sqrt{n}} \right\}$

4. Determine if each of the following series converges or diverges. Justify your answer in each case.
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$
   (b) $\displaystyle\sum_{n=1}^{\infty} 2^{(1/n)}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n! n!}{(2n)!}$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 4n}{n^4}$

5. Find the radius of convergence of the power series
$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n 2^n}.$$

(continued on the back)

6. Determine if the following integrals converge or diverge

(a) $\displaystyle\int_1^\infty \frac{e^{-x}}{1+e^{-x}}\,dx$

(b) $\displaystyle\int_{-1}^1 \frac{1}{x^2}\,dx$

7. Using the Maclaurin series for $\sin x$, compute the Maclaurin series for $f(x) = x^3 \sin 2x$. Write your answer in summation notation.

8. (a) Compute the third Taylor polynomial, $T_3(x)$, for $f(x) = \sqrt{x}$ near $a = 1$.

   (b) If you use your answer in part (a) to estimate $f(x)$ on the interval $[1, 1.2]$, estimate the maximum error that can result.