

MATH 120 — Final Exam — 16 May 2025

Name: _____

Class Section: _____

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Important Information:

WAIT until you are told to start.

For the counting questions, leave your answers in terms of $n!$, $P(n, k)$, $\binom{n}{k}$, and $\binom{n}{k}$.

You are expected to SHOW YOUR WORK in all answers.

Wrong answers provided without work will receive no credit.

Every part of every question is worth the same number of points.

You may use the back of pages if you need more space or for scrap work.

**If you continue work in a different location and want it to be graded,
indicate CLEARLY where it is.**

1. Let B be the set of all **binary strings**.

(a) Give two examples of objects in B and one example of an object not in B . In your answer label which ones are which.

(b) Is B finite or infinite? Explain your reasoning.

2. Let E be the set of even integers and let F be the set of integers divisible by 4.

(a) Write F in roster notation.

(b) Which of the following expressions are True? Circle all that apply.

$$E \subseteq F \quad E \subset F \quad E = F \quad F \subseteq E \quad F \subset E$$

3. Let $X = \{1, 2, 3\}$. Write $X \times \{4\}$ in set-builder notation.

4. Consider all computer code ever written. We will say that a piece of code is **electric** if it is BOTH written using Python AND written using Generative AI.

Use de Morgan's laws to describe **pieces of code that are NOT electric**.

Complete the following sentence **using** the word "or" and **not using** the word "and".

Pieces of code that are not electric ... _____
_____ .

5. Let A be the set of all animals. Let D be the set of dogs, C be the set of cats, and F be the set of animals with fur. Assume dogs and cats are animals with fur.

(a) Draw a Venn diagram representing the relationships between these sets.

(b) Shade the region representing animals that have fur but are not cats.

(c) Write a set expression using A , D , C , and F for the set of animals that are neither dogs nor cats. Your answer does not need to include all four sets.

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6. A standard deck of 52 cards has 4 suits (clubs, diamonds, hearts, and spades) and 13 denominations (A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2).
- (a) How many different 5-card hands are possible?
 - (b) How many 5-card hands consist of cards **all from the same suit**?
 - (c) How many 5-card hands consist of cards **all from different suits**?
7. A bakery sells 6 different types of muffins, including chocolate chip and blueberry.
- (a) In how many ways can you choose 12 muffins **of any flavor** to take home?
 - (b) In how many ways can you choose 12 muffins to take home if you want **at most two** chocolate chip muffins?
 - (c) In how many ways can you choose 12 muffins to take home if you **don't want any** blueberry muffins and you want **at least two** chocolate chip muffins?
8. Consider all permutations of the letters $\{A, B, C, D, E\}$.
- (a) How many permutations start with the letter 'A'?
 - (b) How many permutations contain the sequence 'ABC' in order, somewhere in the word?

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9. Let W be the set of all words in the English language, and let $V = \{a, e, i, o, u\}$ be the set of vowels. Define a function $f : W \rightarrow V$ that takes as input a word and outputs its first vowel.
- (a) Give three examples of words in W and their corresponding outputs from f .
- (b) Is f a well-defined function? Explain why or why not.
10. Consider a function d that takes as input every integer greater than or equal to two and outputs its **smallest** prime divisor.
- (a) What is the domain of this function?
- (b) What is the range of this function?
- (c) What is $d(18)$?
- (d) Describe in words the set $d^{-1}(3)$.
11. You are going to create a function $f : X \rightarrow Y$ that has the following properties.
- The domain X that has exactly five elements.
 - The function f is injective.
 - The function f is not surjective.
- (a) Define your function using any clear method (e.g., listing pairs, arrow diagram, etc.)
- (b) Explain why your function satisfies all three conditions.
12. Compute $\left\lceil \frac{4.5 + \lceil 100.01 \rceil}{3} \right\rceil$. Show every step of your work.

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13. Compute $\log_9(3^{2^5})$. Your final answer should be a number. Explain your steps.
14. Rewrite $\frac{x \ln 3 + y \ln 4}{5 \ln z + 2 \ln z}$ into an expression that uses as few logarithms as possible.
15. Consider a pond containing algae plants that cover more and more of the pond surface every day. Suppose that the pond starts off with $a_0 = 16$ square feet of algae today (day zero) and the amount of algae **increases by 50%** every day.
- (a) Determine the amount of algae on the pond over the next three days: a_1 , a_2 , and a_3 .
- (b) Determine a formula for a_n , the amount of algae on the pond on the n -th day.
16. Write $\left\lfloor \frac{11}{10} \right\rfloor \cdot \left\lfloor \frac{12}{9} \right\rfloor \cdot \left\lfloor \frac{13}{8} \right\rfloor \cdot \dots \cdot \left\lfloor \frac{20}{1} \right\rfloor$ in product notation.
17. Compute $\sum_{i=0}^{100} (1 + 5i)$ using techniques from class. Do not use summation notation in your answer.
18. Rewrite $\log_2 \left(\prod_{i=1}^{10} x^i \right)$ in the simplest form possible, using only **one** logarithmic term that **does not have** exponents and **does not use** product notation.

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19. Suppose that the first day of the year (January 1) is a Tuesday. On what day of the week is the 200th day of the year? Show your work.
20. Calculate $7^{48} \bmod 9$ using the technique of repeated squaring.
21. Use the Euclidean Algorithm to find the greatest common divisor of 510 and 187.
22. Consider the number 720.
- (a) Find the prime factorization of 720.
- (b) How many divisors does 720 have? Use your answer to part (a); do not list them all.
- (c) List all the divisors of 720 that are also multiples of 72.

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