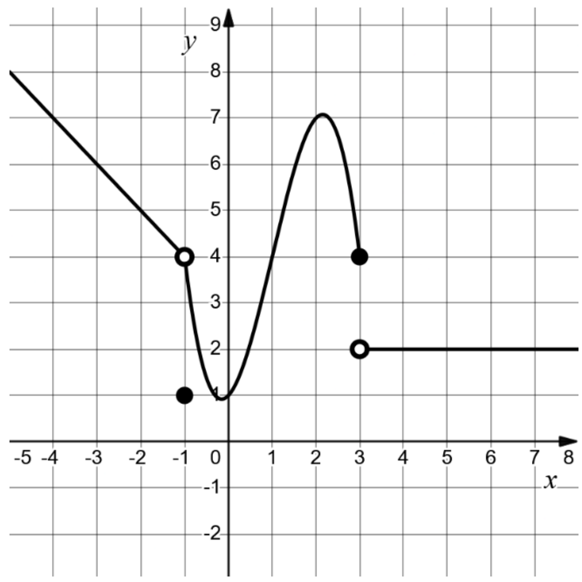
QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 131 **Spring 2025**

Instructions: Answer all questions and show your work in the blue book provided.

1) Use the provided graph of $f(x)$ to evaluate each of the following, if it exists.
   a. $\lim_{x\to -1^-} f(x)$
   b. $\lim_{x\to -1^+} f(x)$
   c. $\lim_{x\to -1} f(x)$
   d. $\lim_{x\to 3^-} f(x)$
   e. $\lim_{x\to 3^+} f(x)$
   f. $\lim_{x\to 3} f(x)$
   g. $\lim_{x\to \infty} f(x)$
   h. $\lim_{x\to -\infty} f(x)$
   i. $f'(3)$



2) Evaluate each of the following limits without using a table or graph:
   a. $\lim_{x\to 8}\dfrac{\sqrt{2x}-4}{x-8}$
   b. $\lim_{x\to 3}\dfrac{9-x^2}{x^2+2x-15}$
   c. $\lim_{x\to 1^+}\dfrac{\ln(x)}{x-5}$
   d. $\lim_{x\to \infty}\dfrac{1+2x^3-5x^5}{8x^5+6x^3}$

3) Given the function $f(x)=5x-x^2$,
   a. use the limit definition of the derivative to find $f'(x)$.
   b. find an equation of the tangent line at the point $(2,6)$.

4) Find $\frac{dy}{dx}$ for each of the following. (You do not need to simplify.)
   a. $y=\dfrac{5x}{3x^3}-\dfrac{4}{\sqrt[7]{x}}+\sqrt[3]{5x^4-x}+\dfrac{e^{5x}}{2x}$
   b. $y=\ln\left(\dfrac{(x^5+2x)^3(2x-1)^4}{e^{x^6+7x}}\right)$
   c. $y=(2x^3-x+8)^{e^{-x}}$, (e^{-x} is an exponent)
   d. $4x^6y-\sqrt[5]{y^2}=\ln 2-x^5$

(continued on the back)

5) Let $f(x) = x^4 - 8x^2 + 7$. Using calculus (not the calculator)

a. find the intervals of increase and decrease of f.
b. find the relative (local) maximums and minimums of f and their coordinates, if any.
c. find the intervals of upward and downward concavity of f.
d. find the inflection point(s) of f, if any.
e. Graph $f(x)$ using the information from parts a-d and label all relative extrema and inflection points.

6) The demand for x number of units is given by the price function $p(x) = 234 - 0.2x$ and the total cost function for x number of units is given by $C(x) = 3x + 19$.

a. Find the total revenue function, $R(x)$
b. Find the total profit function, $P(x)$
c. How many units need to be sold to maximize the profit? What is the maximum profit?

7) The demand function for a certain item is given by $p = 560 - 0.31x^2$, where p is the price in dollars and x is the quantity measured in thousands. If the price is dropping at a rate of \$12.40/week, find the rate at which the quantity is changing when the price is \$529.

8) Suppose the total profit function, measured in dollars, for a company making x number of calculators is $P(x) = -0.000001x^3 - 0.01x^2 + 100x - 5000$ for $0 \le x \le 6000$.

a. Can we use the Intermediate Value Theorem to show that the company will have a profit of \$10,000 for some amount of calculators in the specified range? Explain.
b. Use marginal analysis to approximate the profit from the production of the 51^{st} calculator.
c. Find the actual profit from the production of the 51^{st} calculator.

9) Suppose that \$7500 is invested at the interest rate of 1.75% per year.

a. Compute the balance after 8 years if interest is compounded monthly.
b. Compute the balance after 8 years if interest is compounded continuously.
c. How long will it take your money to double if it is compounded continuously?