Mathematics 142   **SPRING 2025**

Instructions: Answer each question and show your work.

1. Find each of the following.

    a) $\int \left(x + \dfrac{1}{x}\right)^2 dx$

    b) $\int \dfrac{dx}{x(1 + \ln x)}$

    c) $\int \sec^2(2x)\sqrt{1 + \tan(2x)}\ dx$

    d) $\int \dfrac{dx}{\sqrt{9 - 4x^2}}$

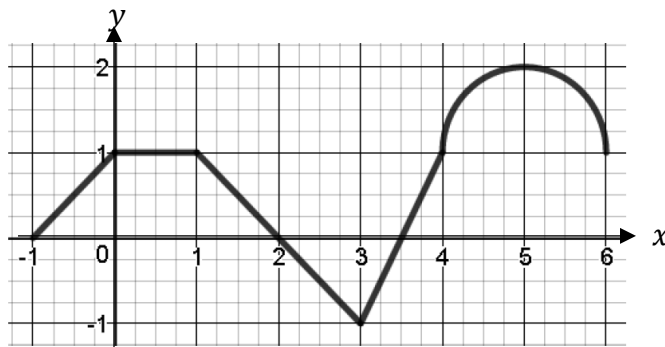   e) $\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{1 + \cos x}\ dx$

2. The graph of $f(x)$ consists of straight lines and a semicircle as shown. Evaluate each integral.

    a) $\int_{-1}^{2} f(x)\ dx$

    b) $\int_{2}^{4} f(x)\ dx$

    c) $\int_{4}^{6} f(x)\ dx$

    d) $\int_{-1}^{6} f(x)\ dx$

3. Consider the integral $\int_0^3 (6x - x^2)\ dx$.

    a)   Estimate the integral using a Riemann sum with 6 subintervals and right endpoints.
    b)   Use the definition of the definite integral as the limit of a Riemann sum to evaluate the integral. (The following formulas may be helpful.)

    $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

    c)   Use the Fundamental Theorem of Calculus Part 2 to evaluate the integral.

4. Find the derivative of each of the following.

    a) $y = 3^{x^3} + \log_3(3x) - \sin^{-1}(\pi x)$

    b) $y = \cos\left[\tan^{-1}(e^x)\right]$

    c) $y = \int_{e^x}^{e^{2x}} \ln t^2\ dt$

    d) $y = (x^2 + 1)^{\ln x}$

(continued on the back)

5. Let R be the region bounded by $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{4}$.

   a) Find the area of the region R.

   b) Find the volume of the solid obtained when region R is rotated about the x-axis. *(One of these identities may be helpful)*

   $$\sin^2 x + \cos^2 x = 1 \qquad \cos^2 x - \sin^2 x = \cos(2x) \qquad \sin^2 x - \cos^2 x = -\cos(2x)$$

   c) Set up, but <u>do not evaluate</u>, the integral that gives the volume of the solid obtained when this region is rotated about the y-axis using the cylindrical shell method.

   d) Set up the integral to find the length of the curve $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{4}$. Then use your calculator to find the length, rounded to 3 decimal places.

6. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{\sin x}{e^{y/2}}$ with initial condition $y(0) = 0$.

7. In a study conducted at University of New Mexico, it was found that the mass (weight) of juvenile desert tortoises exhibited exponential growth after a diet switch. One of these tortoises had a mass of 64 g at the time of the diet switch, and 33 days later, the mass was 73 g.

   a) Find an expression for the mass of the tortoise after t days.

   b) How many days after the diet switch did the tortoise have a mass of 100 g? (Round to the nearest whole number.)