QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 143 **Spring 2025**

<u>Instructions</u>: Answer <u>all</u> questions. <u>Show</u> <u>all</u> <u>work</u>.

1. Evaluate: $\lim_{x \to 0^+} (\cot x)^{\sin x}$

2. Integrate the following:

 a) $\int x \cos(2x)\, dx$

 b) $\int \frac{x^2}{\sqrt{9 - x^2}}\, dx$

 c) $\int \frac{3x^3 + 4x - 6}{x^4 + 2x^2}\, dx$

3. Determine the convergence or divergence of the following improper integrals. If an integral converges, find its value.

 a) $\int_{-\infty}^{0} \frac{e^x}{e^{2x} + 4}\, dx$

 b) $\int_{0}^{3} \frac{2x}{\sqrt[3]{x^2 - 1}}\, dx$

4. Determine if the sequence $\{a_n\}$ converges or diverges. Find its limit if it converges.

 a) $a_n = \left(1 - \frac{6}{n}\right)^n$

 b) $a_n = \left\{\frac{n^5 + 4n - 5}{n^2 - 4n - 5n^3 + 2}\right\}$

5. Determine whether each series is absolutely convergent, conditionally convergent, or divergent.

 a) $\sum_{n=1}^{\infty} \cos\left(\frac{\pi n^2}{6n^2 + 1}\right)$

 b) $\sum_{n=1}^{\infty} \frac{n^2}{6n^4 - 5}$

 c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$

(continued on the back)

6. Find the exact sum for each of the following series:

a) $\sum_{n=0}^{\infty} \frac{3^n}{5^n \cdot n!}$

b) $\sum_{n=1}^{\infty} \frac{2+4^n}{6^n}$

c) $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$

7. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{8^n \cdot n^4}$.

8. Given $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.

a) Find the Maclaurin series representation for the function $f(x) = x^2 e^{-x^2}$.

b) Use the result of a) to obtain a series representation for $\int_0^{1/2} x^2 e^{-x^2}\, dx$. Then use the fewest number of terms to estimate its value with an error less than 0.0001.

9. Let $f(x) = x^{3/2}$.

a) Write the fourth degree Taylor polynomial, $T_4(x)$, for $f(x)$ centered at $a = 4$.

b) Use Taylor's formula to estimate the accuracy when $T_4(4.2)$ is used to approximate $f(4.2)$.