

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION

$2\frac{1}{2}$ Hours

Math 151

Spring 2025

Instructions:

- Answer all questions and show all work in the provided Blue Book.
- Partial credit will be awarded for relevant work, but no credit will be given without supporting work. Maximal credit is only assigned to well-written solutions that demonstrate algebraic proficiency and conceptual understanding.
- Calculators are permitted per the department's policy. No notes, programs, internet apps, or other devices are allowed.
- Unless otherwise stated, answers must be exact (e.g., $\frac{1}{3}$ rather than 0.333).

- 1.** Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$, or does not exist, the work you show must support your answer.

a) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 - 3x - 4}$

b) $\lim_{x \rightarrow 1^-} \frac{|2x - 3|}{x - 1}$

c) $\lim_{x \rightarrow \infty} \frac{1 + 8x - 7x^2}{4x^2 - 3}$

d) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(2x)}$

2. Let $f(x) = \frac{x}{x + 8}$

- a) Using the definition of the derivative, find $f'(x)$.
- b) Write an equation of the tangent line to the graph of f at $x = 4$.

3. Let $g(x) = \begin{cases} \frac{\sin^2(4x)}{x^2}, & x \neq 0 \\ c, & x = 0 \end{cases}$

Find the value of c for which $g(x)$ is continuous at $x = 0$.

- 4.** Compute $\frac{dy}{dx}$ for each of the following functions. Do not simplify.

a) $y = \frac{3 \tan(2x)}{(6x^3 + 5x^2 + x)^3}$

b) $y = \sin^3(x^7 + 4)$

c) $x^3 y^2 - 3x^3 + y^5 = -1$

d) $y = \int_{x^2}^9 \sqrt{t^7 + \pi} \, dt$

(Continued on the back)

5. Let $h(x) = \frac{2x^2}{x^2 - 25}$. Use Calculus,
- find and state all asymptotes of the graph of h , if any.
 - find and state all intervals on which h is increasing and on which h is decreasing.
 - find and state all relative extrema, if any.
 - find and state all intervals on which the graph of h is concave up and on which the graph of h is concave down.
 - find and state any inflection points on the graph of h , if any.
 - Using the information found above, sketch the graph of h .
6. Person A lives on Jewel Avenue 1000 yards from the corner of Jewel Avenue and Main Street. Person B lives on Main Street 1100 yards from the same corner. Jewel Avenue and Main Street are perpendicular to each other. Each person leaves his home at 1:00 PM and runs to the corner of Jewel and Main at the rate of 80 yards per minute. How fast is the distance between them changing at 1:05 PM? Round your answer to the nearest whole number.
7. Two nonnegative numbers add up to 72. Find the maximum possible value of the sum of their square roots.
8. Show, without the use of a calculator, that $y = x^5 + x - 1$ has exactly one real root.
9. Use the limit of a Riemann sum to evaluate $\int_0^3 (5x - 2) dx$.
- Note: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$
10. Evaluate the following integrals.
- $\int_1^2 (6x^2 + 2x + 1) dx$
 - $\int_1^9 \frac{x-2}{\sqrt{x}} dx$
 - $\int (\tan(3x) + 1)^5 \sec^2(3x) dx$