MATH 120 — Final Exam — 16 December 2025

Name: _____

Section: _____

CUNYfirst Number: _____

Important Information:

WAIT until you are told to start.

Write your name at the top of EVERY PAGE.
Write your solutions in the boxes provided.
If you continue work in a different location and want it to be graded, indicate CLEARLY where it is.

For the counting questions, leave your answers in terms of $n!$, $P(n,k)$, $\binom{n}{k}$, and $\left(\binom{n}{k}\right)$.

You are expected to SHOW YOUR WORK in all answers.
Wrong answers provided without work will receive no credit.
Every part of every question is worth the same number of points.

Name: _____

1. Consider the mathematical symbol $\varnothing$.

   (a) Translate the meaning of this symbol into English.

   (b) Explain why $\{\varnothing\}$ is not the same as $\varnothing$.

2. Let $S = \{x \in \mathbb{N} \mid \sqrt{x} < 10\}$.

   (a) Write S in roster notation.

   (b) Is $\varnothing \subseteq S$? Explain.

3. Write the set $V = \{4, 9, 16, 25, \ldots\}$ in set-builder notation.

Name: _____

4. Define the set $T = \{a, b\}$.

   (a) List all elements of $T \times T \times T$.

   (b) List all **proper subsets** of T.

5. A travel agency defines a "Dream Vacation" to be a trip that is both "inexpensive" and "tropical". Why is the statement "A vacation that is expensive and cold" an **incorrect** description of all vacations that are not Dream Vacations?

6. (a) **Draw and label** a Venn diagram for three sets, A, B, and C.
   Shade the region representing $(A \cap B) \cup C$.

   (b) If A is the set of Computer Science majors, B is the set of Mathematics majors, and C is the set of Chemistry majors, how would you describe in words the region you shaded in part (a)?

Name: _____

7. You are distributing 20 coins into 4 distinct piggy banks.

   (a) In how many ways can you distribute 20 **different** coins into the 4 distinct piggy banks?

   (b) In how many ways can you distribute 20 **identical** coins into the 4 distinct piggy banks, if each piggy bank must receive at least one coin?

   (c) Is it possible for each piggy bank to have a different number of coins? Explain your answer.

8. The student combinatorics club has 20 students.

   (a) In how many ways can the club choose its club officers: one president, one vice-president, and one treasurer?

   (b) In how many ways can the club form a committee of 5 members if at least one of its club officers must be on the committee?

   (c) In how many ways can the club break into two teams of 10 dodgeball players?

Name: _____

9. You have 5 different books and 3 different plants that you want to arrange on a shelf.

   (a) How many arrangements are there of the books and plants on this shelf?

   (b) If the books must stay together and the plants must stay together, how many arrangements are there now?

10. Let $f : \mathbb{Q} \to \mathbb{Z}$ be defined by $f(p/q) = p + q$. Is f well-defined? Explain why or why not.

11. Let $g : \mathbb{N} \to \mathbb{N}$ be defined by $g(k) = k \bmod 7$.

   (a) Is g injective?

   (b) Is the codomain of g equal to the range of g? Explain your answer.

Name: _____

12. Let M be the set of months in the year. Suppose that m is a month. Define $h(m)$ to be the number of days in m in the year 2025.

    (a) What is a possible codomain for h? Explain your reasoning.

    (b) What is $h(\text{December})$?

    (c) What is $h^{-1}(35)$?

13. Give a counterexample to the statement: $\lfloor x \cdot y \rfloor = \lfloor x \rfloor \cdot \lfloor y \rfloor$ for all $x, y \in \mathbb{R}$.

14. (a) Convert $\dfrac{(x^2y)^3}{x^{-1}y^4}$ into an expression where x and y each appear **once** and only contains **positive** exponents.

    (b) Convert $3\log_2(x) - \log_2(x^2) + \log_2(4y)$ into an expression involving a single logarithm.

Name: _____

(c) Convert $e^{2\ln 5}$ into an expression that does not include logarithms.

(d) Convert $\log_{10}(16)$ into an expression involving $\log_2$.

15. Consider the sequence defined by $a_n = (-1)^n \cdot n$.

(a) List the first 5 terms starting with $n = 0$.

(b) Is this an arithmetic sequence? Explain your reasoning.

16. Consider the series $6 + 12 + 24 + \cdots + 3072$.

(a) Write this series in sigma notation.

(b) Determine the sum of this series.

Name: _____

17. Consider the product $\prod_{k=2}^{19}(1-\frac{1}{k})$.

(a) Write this product as a product of factors, each of which is a fraction of the form $\frac{p}{q}$ for $p, q \in \mathbb{Z}$. Include **at least five** factors of this type. You may use ellipses "$\cdots$" but do not use a $\prod$.

(b) Determine the value of this product when all terms are multiplied together.

18. Let $S = \sum_{k=1}^{n} \log_{10}(k+1)$.

(a) Write S as an expression involving a product instead of a sum.

(b) Calculate 10^S when $n = 5$.

Name: _____

19. What is the remainder when $100! + 17$ is divided by 11? Explain.

20. Planet A orbits a star every 16 years. Planet B orbits every 12 years. If the planets are aligned now, in how many years will they be aligned again?

21. Use the Euclidean Algorithm to find the greatest common divisor of 1794 and 520.

22. Is $2^3 \cdot 3^2 \cdot 5$ a divisor of $2^4 \cdot 3 \cdot 5^2$? Explain why. Include a discussion about exponents.