Name:_____ **CUNY ID:**_____

Fall 2025: Math 122 Final Exam

Instructions:

- Calculators are **not** permitted for use.
- Maximum credit is only assigned to well-written solutions that **demonstrate** mathematical proficiency and conceptual understanding. Illegible work will be ignored.
- There are questions on the front **and** back of each page.

1. The parts for this question are **unrelated**.

a) Let $f(x) = x + 5$ and $g(x) = \sin(x)$. (3 points)

- Find $(f \circ g)(x)$.
- State the range of $(f \circ g)(x)$ using interval notation.

b) Expand the following expression. (4 points)

$$\log_4\left(\frac{x^5}{\sqrt{x^2+2}}\right)$$

2. Consider the one-to-one function $h(x) = \ln(x + 2)$. (4 points)

- Use algebra to find the inverse function, $h^{-1}(x)$.
- State the domain and range of $h^{-1}(x)$ using interval notation.

3. Let $f(x) = 2x^2 - 12x + 23$

a) Use algebra to express $f(x)$ in vertex form. **Then** state the coordinates of the vertex for the function. (5 points)

b) State the range for $f(x)$ in interval notation. (2 points)

c) State the intervals of increase and decrease for $f(x)$ in interval notation. (2 points)

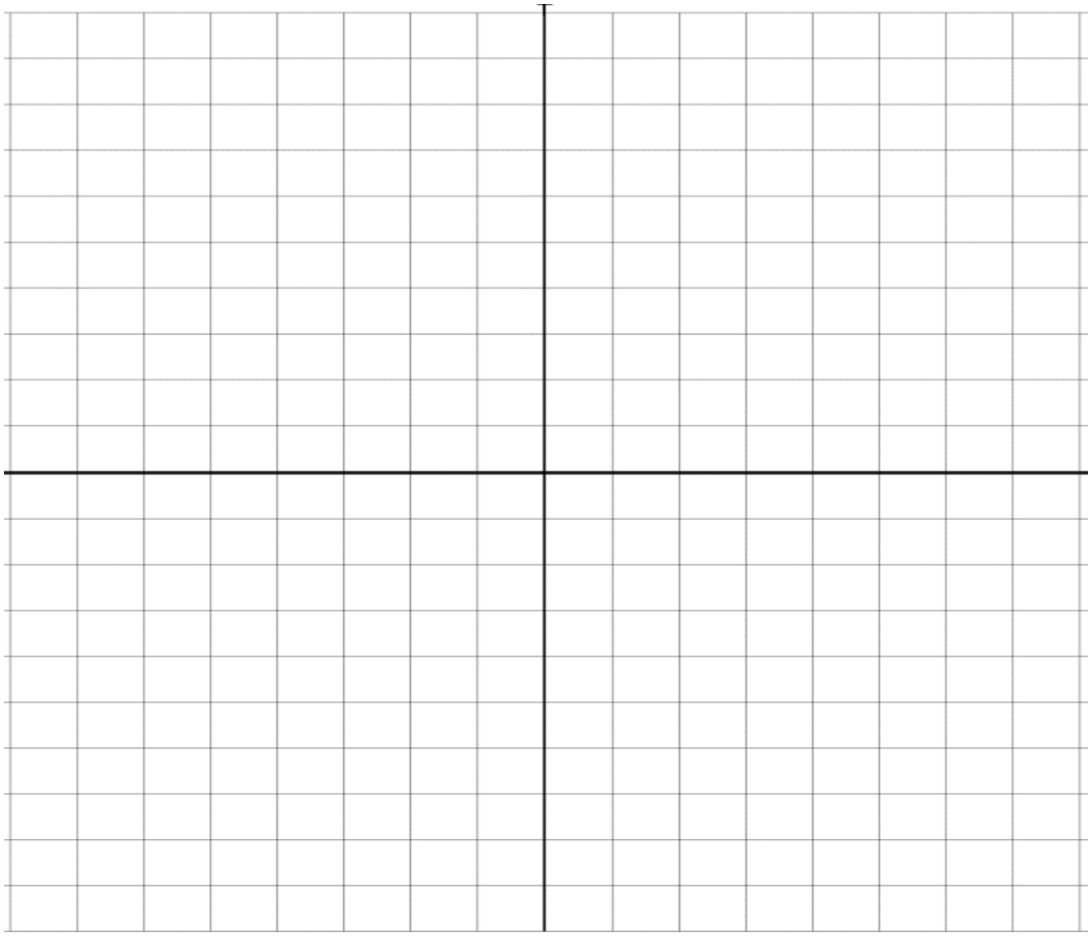
d) Use algebra to find and simplify the difference quotient, $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$. (5 points)

4. Let $f(x) = \frac{3x-5}{x+1}$.

a) Find any vertical and horizontal asymptotes by their equation of $f(x)$. (4 points)

b) Use algebra to find all x- and y- intercepts of $f(x)$. **Also**, find one additional point (x, y) on the graph. (5 points)

c) Sketch $f(x)$ on the provided set of axes below. Make sure to **label** all asymptotes by their equation **and** any points you found in part **b)** above. (6 points)



d) Write the domain and range of $f(x)$ in interval notation. (4 points)

5. Use algebra or any appropriate mathematics to find the domain of $g(x) = \log(2 + x - x^2)$. Write the domain using interval notation. (5 points)

6. Evaluate $\cot\left(\sin^{-1}\left(\frac{3}{10}\right)\right)$. Give an exact answer in simplified form. (4 points)

7. Consider the following piecewise defined function.

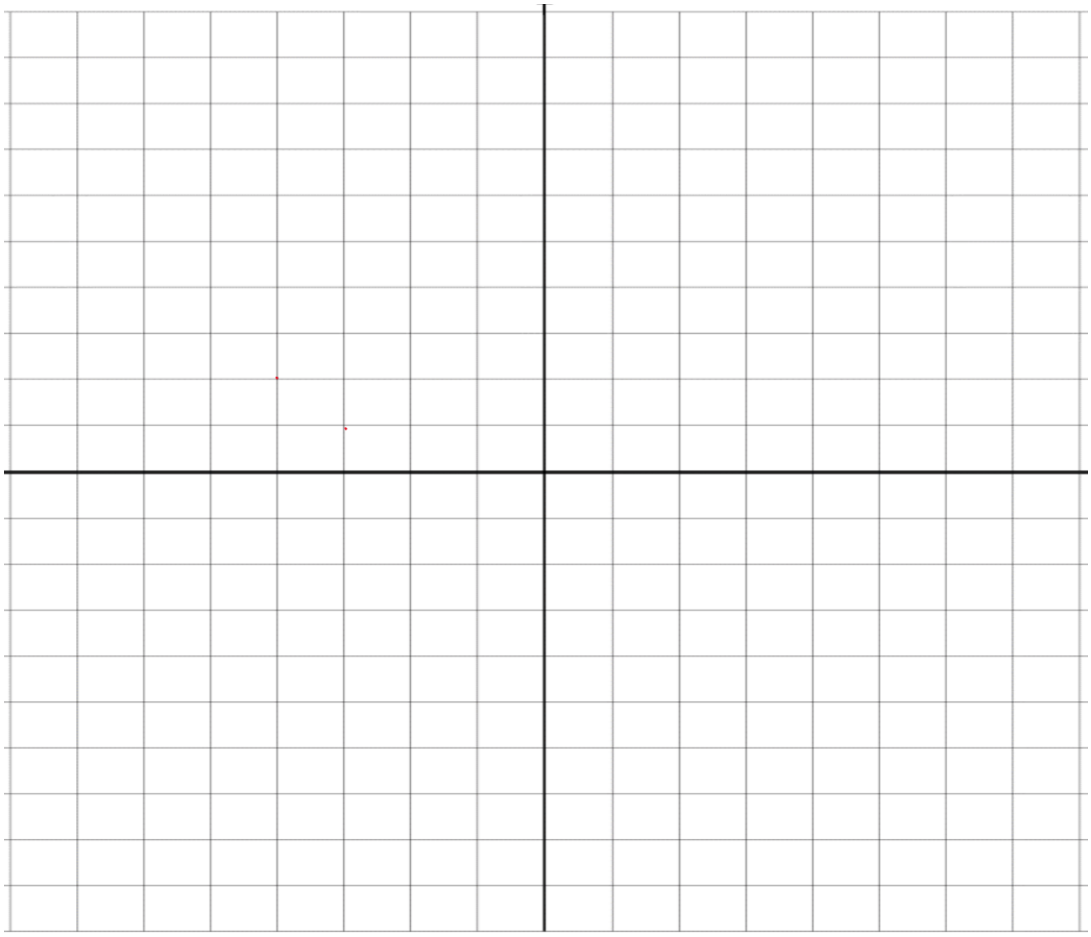
$$f(x)=\begin{cases}|x|-2 & \text{if } x<-2\\ x-2 & \text{if } -2\le x<2\\ \log_2(x-2) & \text{if } x>2\end{cases}$$

a) Evaluate $f(-6)$, $f(-2)$, $f(2)$, and $f(6)$. If the value is undefined, write DNE. (1+1+1+1 points for a total of 4 points)

b) Find any vertical and horizontal asymptotes by their equation of $f(x)$. (2 points)

c) Sketch the graph of $f(x)$ either by using transformations or by making a table of values. (9 points)

- **Plot and label** any/all intercepts.
- **Plot and label** the points associated with the values you found in part **a)**.
- **Sketch and label** any asymptotes by their equation.



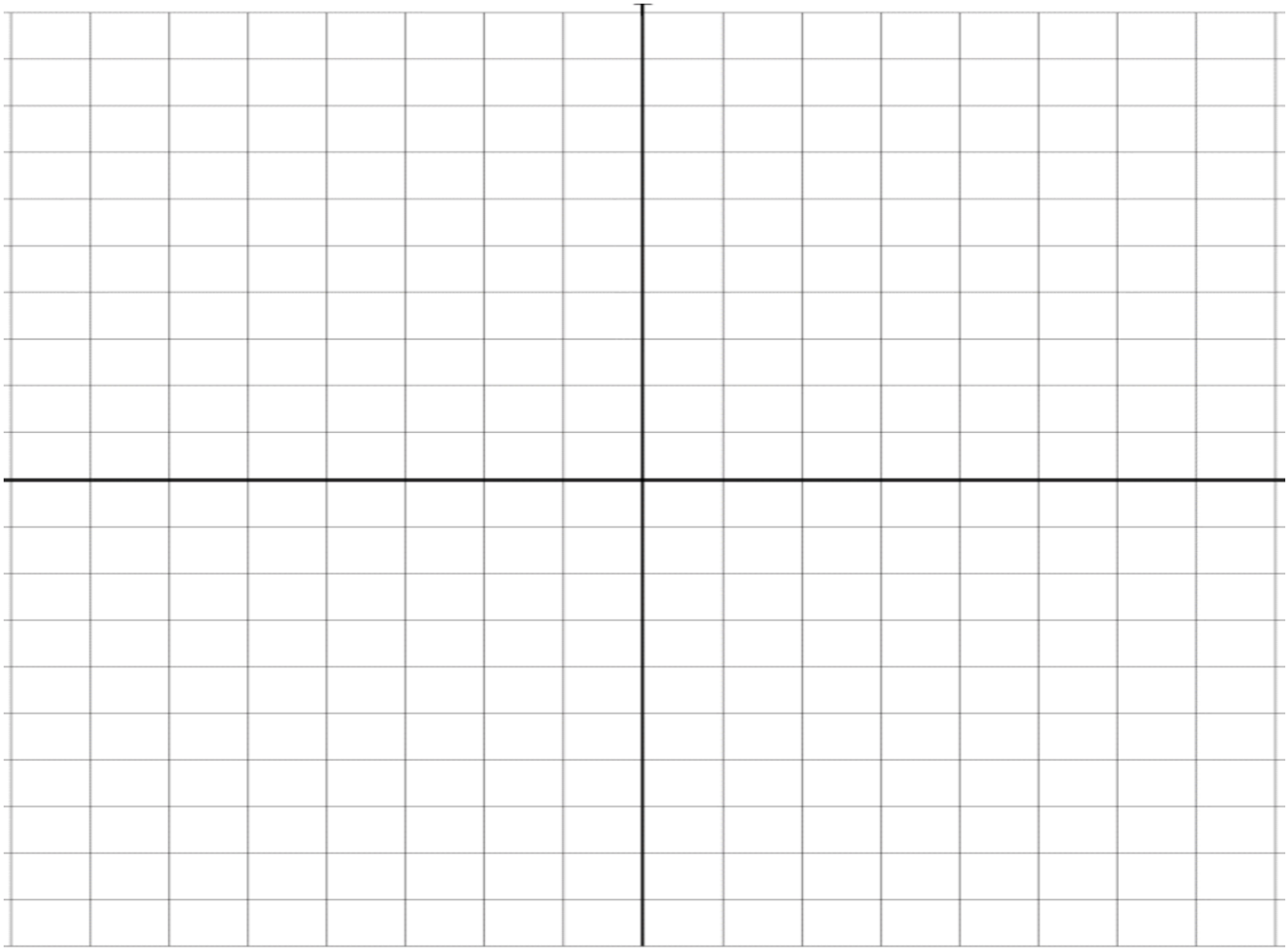
d) State the domain and range of $f(x)$ in interval notation. (3 points)

8. Consider the polynomial function $f(x) = 4x^3 - 7x + 3$.

a) Use algebra and applicable facts and theorems of polynomials to write $f(x)$ in fully factored form. (5 points)

b) State the end behavior of $f(x)$. (2 points)

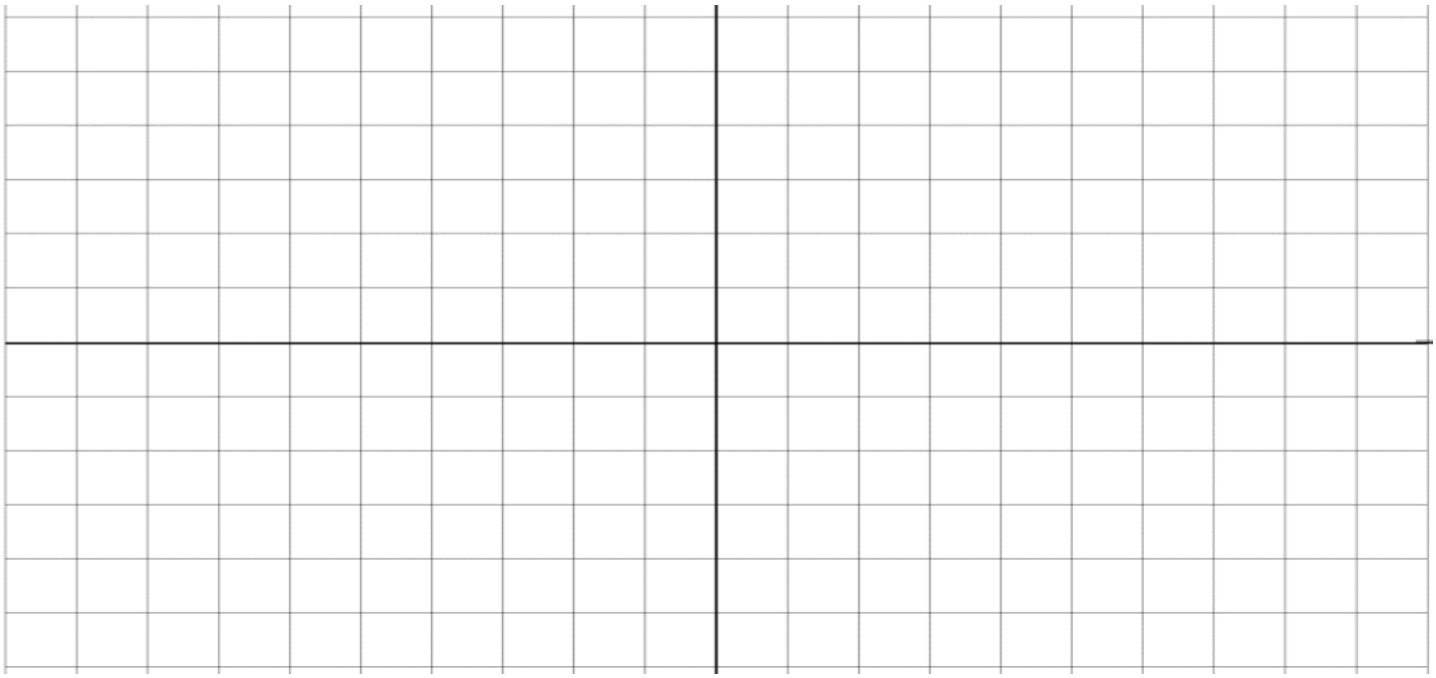
c) Sketch a reasonably accurate graph of $f(x)$ on the provided set of axes below. Make sure to **label** all x- and y- intercepts. (5 points)



9. Consider the function $f(x) = \cos\left(3\left(x + \frac{\pi}{6}\right)\right)$

a) Using your knowledge of trigonometric functions, find the amplitude, period, and the equation of the midline of $f(x)$. (3 points)

b) Sketch the graph of $f(x)$ for one full cycle. Make sure to label all intercepts. Your sketch should have at least 5 clearly labeled points including any x- and y- intercepts. (6 points)



10. Use algebra and your knowledge of trigonometric identities to find **all** solutions for the equation $\sin(2x) = \cos(x)$ on the interval $[0,2\pi)$. (8 points)