Final Examination
2 ½ Hours

Mathematics 131   **Fall 2025**

Instructions: **Answer all questions.** **Show all work in the provided blue exam book.**

1.   Given $f(x) = \frac{3}{x}$.
   a)   Use <u>the limit definition of the derivative</u> to find $f'(x)$.
   b)   Find an equation of the line tangent to the graph of $f(x)$ at the point $(3,1)$.

2.   Use the provided graph of $f(x)$ to evaluate each of the following:
   a) $\lim_{x \to 2^+} f(x)$

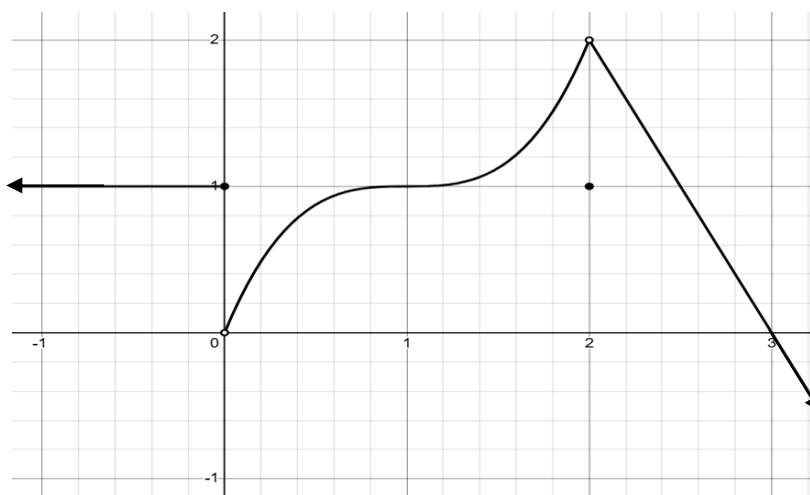   b) $\lim_{x \to 2^-} f(x)$

   c) $\lim_{x \to 2} f(x)$

   d) $f(2)$

   e) $\lim_{x \to -\infty} f(x)$

   f) $f(0)$

   g) $\lim_{x \to 0^+} f(x)$

   h)   For what value(s) of x does the function f have a discontinuity? Explain using the definition of continuity.

3.   Evaluate each limit. ($+\infty$, $-\infty$, and DNE are acceptable possible answers.)

   a) $\lim_{x \to -1} \dfrac{x^2 - 1}{x^2 + 4x + 3}$

   b) $\lim_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

   c) $\lim_{x \to +\infty} \dfrac{3 - x^3 + 10x^2}{5x + 2x^3}$

   d) $\lim_{x \to 0^-} f(x)$, given $f(x) = \begin{cases} x^2 + 10 & , \text{ if } x < 0 \\ \sqrt{x} & , \text{ if } x \geq 0 \end{cases}$

4.   Given $f(x) = x^3 - 2x^2 + 3$.
   a)   Can the Intermediate Value Theorem be used to show $f(x) = 1$ for some value of x in the interval $0 \leq x \leq 1$? Justify your answer.
   b)   Find the absolute extrema of $f(x)$ on the interval $0 \leq x \leq 1$.

5.   Find $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify.

   a) $y = \frac{4x^5}{5} + \sqrt[3]{x^5} - 2e^x + \ln(2x^4 + x) - \ln(x^7)$

   b) $y = \frac{e^{3x-1}}{(5x^2 + x)^{100}}$

   c) $y = 5^{x^2+3x}$ (Hint: Use logarithmic differentiation.)

   d) $5x^2y + y^3 = 2x^3 + 15$

6.   a)   Suppose $5000 is invested into a bank that pays an annual interest rate of 3.5%.
            Compute the balance after 6 years if interest is compounded
            i.   monthly.
            ii.   continuously.

      b)   How much money should be invested at an annual interest rate of 2% compounded
            continuously so that it will be worth $1000 in five years?

7.   Let $f(x) = 2x^3 - 6x^2 + 5$. Using calculus (not your graphing calculator):
      a)   Find the intervals of increase and decrease of f.
      b)   Find the relative (local) maxima and minima of f and their coordinates.
      c)   Find the intervals of upward and downward concavity of f.
      d)   Find the inflection point(s) of f and their coordinates.
      e)   Using the information from parts a) – d), carefully sketch the graph of f. Label all
            relative extrema, inflection points, and the y-intercept.
      Be sure to clearly label each part of the problem you are working on

8.   A manufacturer will produce and sell x units of a product when the price is
      $p = 3x^2 - 4x$ dollars. The total cost to produce x units is given by the function
      $C(x) = -.05x^3 + 90x + 1000$ dollars.
      a)   Find the total revenue function, $R(x)$.
      b)   Find the total profit function, $P(x)$.
      c)   Use marginal analysis to estimate the **cost** of producing the 7th unit.
      d)   Find the actual **cost** of producing the 7th unit.
      e)   Find the average **cost** per unit if 7 units are produced.

9.   An ice sculpture is melting into an approximately circular puddle forming at its base. The area
      of the puddle is increasing at the rate of $10\pi \ ft^2$ per hour. Find the rate that the radius of the
      puddle is changing at the instant when the radius of the puddle is $2 \ ft$.