QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 142 **Fall 2025**

<u>Instructions:</u> <u>Answer all questions.</u> <u>Show all work.</u>

1. Find the derivative of each of the following (algebraic simplification not necessary):

   a. $f(x) = \dfrac{e^{\sqrt{x}}}{x}$

   b. $f(x) = x^2 \ln(\cos x + \tan^{-1} x)$

   c. $f(x) = \sin^{-1}(x^3 + 1)$

   d. $f(x) = \displaystyle\int_1^{2x} t \sin(t^2)\, dt$

2. Find y', the derivative of y, using logarithmic differentiation: $y = \dfrac{(x^2 - 1)\sqrt{2x + 5}}{\tan^2 x}$

3. Evaluate each of the following integrals:

   a. $\displaystyle\int \frac{x^3 + 2x^2 - x + x^2 e^x}{x^2}\, dx$

   b. $\displaystyle\int \frac{4x}{(2x^2 + 3)^3}\, dx$

   c. $\displaystyle\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

   d. $\displaystyle\int \frac{\sqrt{\ln x}}{x}\, dx$

   e. $\displaystyle\int \frac{e^{-\frac{1}{x}}}{x^2}\, dx$

   f. $\displaystyle\int \frac{x^2}{1 + x^6}\, dx$

4. Find the average value of the function $f(x) = \sin 4x$ on the interval $[0, \pi]$.

5. A bacteria culture initially contains 10,000 cells. After two hours, the number of bacteria has grown to 60,000. Assume that the culture grows exponentially.
   a. Find a model for the number of bacteria present after t hours.
   b. Use your model to predict how many bacteria will be in the culture at the end of five hours.
   c. How long will it take for the bacteria population to reach 500,000?

6. Let R be the region bounded by the curves $y = x^3$ and $y = 2x - x^2$, where $x \geq 0$.
   a. Find the area of the region R.
   b. Find the volume of the solid obtained by rotating R about the x-axis.
   c. Find the volume of the solid obtained by rotating R about the y-axis.

7. Find the solution of the differential equation $\frac{dy}{dx} = xe^{-y}$ that satisfies the initial condition $y(0) = -1$.