QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 143 **Fall 2025**

Instructions: Answer all questions. Show all work.

1. Compute each of the following integrals:

   a) $\int x^2 e^{5x}\, dx$

   b) $\int \sin^3 x \sqrt{\cos x}\, dx$

   c) $\int \frac{x^3}{\sqrt{x^2+9}} dx$ (using trigonometric substitution)

   d) $\int \frac{6x^2+3x+2}{x^3+2x^2} dx$

2. Without using your calculator, find each of the following limits:

   a) $\lim_{x\to 0} \frac{\cos x - 1}{e^x - x - 1}$

   b) $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{4n}$

3. Determine if each of the improper integral converges or diverges. If it converges, find its value.

   a) $\int_{-\infty}^{\infty} \frac{x}{(x^2+1)^2} dx$

   b) $\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$

4. Determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n+1}$.

5. Determine if the following series are absolutely convergent, conditionally convergent or divergent. Justify your answer.

   a) $\sum_{n=0}^{\infty} \frac{2n^2-n+10}{3n^3-4n+2}$

   b) $\sum_{n=0}^{\infty} \frac{2^n \sin^2 n}{5^n}$

   c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

   d) $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{(n+1)!}$

(continued on the back)

6. a) Use the Maclaurin Series for $\cos x$ to find the Maclaurin series for $f(x) = x^3 \cos x$.

b) Using your answer to part a), evaluate $\int_0^1 x^3 \cos x \, dx$, correct to 4 decimal places.

7. Use the power series for $\frac{1}{1-x}$ to find a power series representation for $f(x) = \frac{1}{4+x^2}$.

8. a) Find $T_4(x)$, the fourth Taylor polynomial, for $f(x) = \ln\sqrt{x}$ about $a = 1$.

b) Use your answer to part a) to approximate $\ln\sqrt{1.1}$.

c) Estimate the maximum error that can result when using your approximation in part a) to approximate $f(x)$ on the interval $0.9 \leq x \leq 1.1$.