**QUEENS COLLEGE**
**MATHEMATICS DEPARTMENT**
**Final Exam - Mathematics 151 - FALL 2025**
**Time 2 Hours 30 Minutes.**

**Last, First Name :**_____

**Last 5 digits of your CUNY ID : #**_____

| $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $Q_7$ | $Q_8$ | $Q_9$ | $Q_{10}$ | $Q_{11}$ | **Total** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| 40 | 60 | 45 | 45 | 25 | 30 | 25 | 30 | 30 | 40 | 30 | 400 |

Table 1: Point Values for each Question

**Instructions. Answer all questions in the provided space. Show all your work for full credit. Ti-84 Plus or equivalent Calculators are permitted.**

1. **(Estimated time: 9 minutes)** Evaluate the following limits without using a calculator:

   (a) $\lim_{x\to 3} \frac{x^2-9}{x-3} =$

   (b) $\lim_{x\to 0} \frac{\sin(3x)}{x} =$

   (c) $\lim_{x\to \infty} \frac{3x^2+2x-1}{5x^2-x+4} =$

2. **(Estimated time: 15 minutes)** Find the first derivative for each of the following. Simplify your answer where indicated.

(a) $f(x) = (x^2+1)^3 \cdot (\sin(\tan x))^6$ find f'(x);

(b) $g(x) = \frac{\sec x}{x^2+2x+1}$ find g'(x);

(c) $(x^2+y^2)^2 = 18(x^2-y^2)$ find $\frac{dy}{dx}$;

(d) $F(x) = \int_1^{x^2} \frac{t}{t+1}\, dt$. find F'(x) [Simplify your answer];

3. **(Estimated time: 16 minutes)** Evaluate the following integrals using the substitution rule:

   (a) $\int_1^2 (2x+1)^5\,dx$

   (b) $\int 3x\cos(x^2+1)\,dx$

   (c) $\int x^2\sec^2(x^3)\cos(\tan(x^3))\,dx$

4. **(Estimated time: 12 minutes)** Find the indicated primitive function of:

   (a) $f(x) = 4x^3 - 2x + 5;$ Find $F(x)$ if $F(1) = -2$

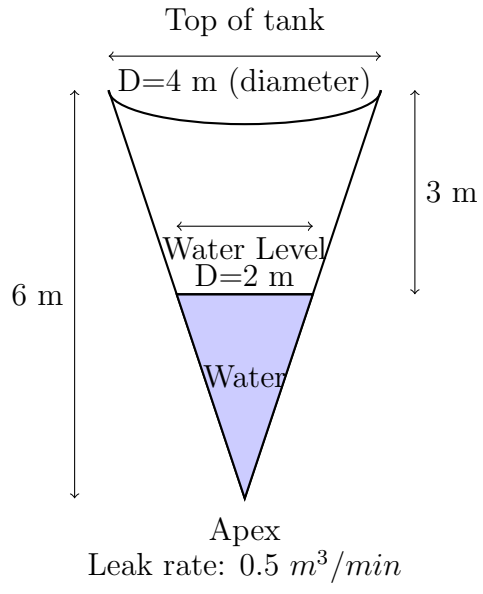
(b) $g'(x) = \sin x + \cos x + \frac{1}{x^2}$; Find $g(x)$

(c) $h''(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$; Find $h(x)$ if $h'(1) = 0, h(1) = 1$

5. **(Estimated time: 10 minutes)** Use linear approximation to estimate $\frac{2}{99}$. Provide the linear approximation formula used for your computation.

**Hint:** You may want to use similar triangles.

6. **(Estimated time: 12 minutes)** Water is leaking out of an inverted conical tank at a rate of 0.5 cubic meters per minute. The tank has a height of 6 meters and a base radius of 2 meters. How fast is the water level dropping when the water is 3 meters deep?




Leak rate: 0.5 $m^3/min$

7. **(Estimated time: 10 minutes)** For $g(x) = x^2 + 1$ on $[0, 3]$, find $c$ such that $g'(c) = \frac{g(3)-g(0)}{3-0}$. Interpret geometrically. Justify your answer and indicate the theorem/theorems used at each step.

8. **(Estimated time: 10 minutes)** For the function $f(x) = x^3 - 3x - 1$ on the open interval $(-1, 1)$, does there exist any point $x = c$ where $f(c) = 0$? Is this point unique? Explain why. Justify your answer and indicate the theorem/theorems used at each step.

**Hint:** Volume of a cylinder is base × height. The Area of the side can be drawn as a rectangle where height is one of the sides and the circumference of the base or the top is the other.

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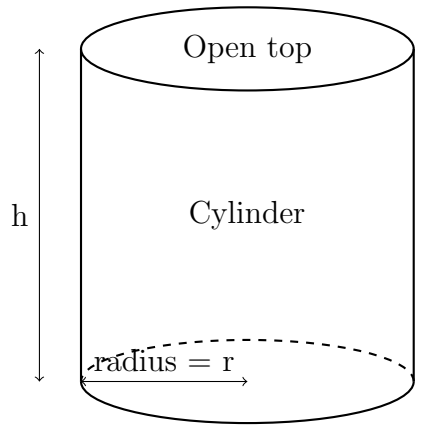
9. **(Estimated time: 12 minutes)** A cylindrical container with no top is to be made from 12 square meters of material. Find the dimensions (radius and height) that maximize the volume. What is the maximum volume?




Total Material: 12 $m^2$ (bottom + side)

10. **(Estimated time: 15 minutes)** Approximate $\int_0^2 (x^2+1)\,dx$ using the midpoint rule with $n = 4$ subintervals. Use the Limit of a Riemann[1] Sum to compute the exact value of the integral.

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[1] [Note: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$]

11. **(Estimated time: 17 minutes)** Analyze the function $\kappa(x) = x^3 - 3x^2 + 2x$:

(a) Find domain, intercepts, and asymptotes (if any).

(b) Find first and second derivatives.

(c) Determine intervals of increase/decrease, concavity .

(d) Identify local extrema (if any) and inflection points (if any).

(e) Sketch the graph based on the above steps, labeling key features.