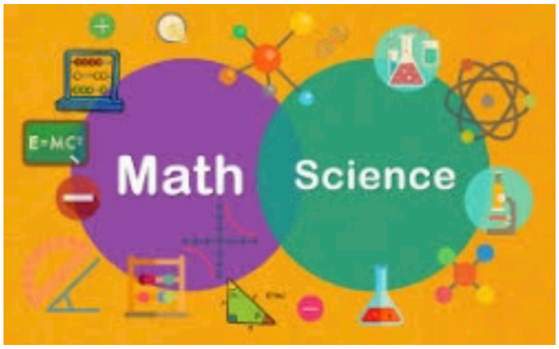

Mathematics Peer Mentor Manual

STEM
Bridges Across Eastern Queens

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Table of Contents

Introduction

Q: What is the **purpose** of this manual?
A: The **objectives** of this manual are to:

- assist peer mentors in their introductory math, or other math intensive courses at Queensborough Community College and Queens College as they work with students who are either having difficulty with the content or who feel they need more support than can be provided by course instructors
- identify the basic math concepts, principles, or operations that students most frequently have difficulty understanding/mastering in their introductory math courses
- identify best practice strategies which mentors can use for each of these concepts/principles/operations

The United States is not producing college graduates with majors in science, technology, engineering, and mathematics (STEM) in sufficient numbers to meet workforce demands. The STEM Bridges Across Eastern Queens project (https://hsistem.qc.cuny.edu) addresses this problem by targeting the disproportionate attrition in STEM among traditionally underrepresented students and also linked to current inefficiencies in transferring from two-year to four-year institutions. The project, funded by a five-year grant to Queens College by the US Department of Education HSI-STEM program, aims to graduate more Hispanic and low-income students with Baccalaureate degrees in STEM and to develop two-year to four-year articulation agreements.

The **Most Frequent Student Misconceptions** in college mathematics and math intensive courses are explained below.

- Each concept will include an algebraic and graphical explanation, where appropriate, as well as an application/activity designed specifically to assist college mathematics students.
- Additionally, unique pedagogical strategies customized to meet students' academic needs have been developed, and are presented throughout each section.

Key of Icons

- Throughout this manual, you will see various icons.
  Please use the below as a key to refer to:

- ❓Question
- ⮕ Definition
- 💡 Answer/Solution
- 🧠 💭 Recall
- 🏃 🎯Activity/Application
- ☑ Check/Verify
- 🗣 Don’t forget to tell / ask students!
- ⚠ Common Student Error/Misconception
- AB Algebraic Explanation
- 🪜 📐 Best Practice Pedagogical Strategy
- 🛑 Stop!
- 🚸 Proceed with Caution!
- 🔬Application to Science Courses
- 🏃 💡 Kinesthetic Aid
- 🔜 Topic will be seen in a forthcoming section of this manual, or in Calculus

SI Prefixes

What is a "metric prefix"?

→ A **metric prefix** is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit.

- Each prefix has a unique symbol that is assigned to the unit symbol.
- For instance, the prefix $milli-$ may be added to meter (m) to indicate division by one thousand $(1\ mm = \frac{1}{1{,}000} m)$.
- It follows that $1\ m = 1{,}000\ mm$.

What are the "SI prefixes"?

→ The **SI prefixes** are metric prefixes that were standardized for use in the International System of Units (SI) by the International Bureau of Weights and Measures (BIPM).

- The BIPM specifies twenty prefixes for the International System of Units (SI).
- The twelve most commonly used SI prefixes in college science courses are: **Tera**, **Giga**, **Mega**, **kilo**, **hecto**, **deca**, **deci**, **centi**, **milli**, **micro**, **nano**, and **pico**.

How can peer mentors help students remember the SI prefixes?

The following mnemonic device can be used to assist students in remembering the SI Prefixes:

The Great Man king henry's daughter betsy
drinks cold milk µntil nine pm.

exponent	**12**	**9**	**6**	**3**	**2**	**1**	**0**	**−1**	**−2**	**−3**	**−6**	**−9**	**−12**
	10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{1}	$10^{0} = 1$	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}
prefix	Tera	Giga	Mega	kilo	hecto	deca	BASE	deci	centi	milli	micro	nano	pico
	T	G	M	k	h	da		d	c	m	µ	n	p

Moving to the **right** from the base (10^0), the power the base 10 is raised to is:
1, 2, 3, and then, multiples of 3 (6, 9, 12, etc.)

Moving to the **left** from the base (10^0), the power the base 10 is raised to is:
1, 2, 3, and then, multiples of 3 (6, 9, 12, etc.)

⚠ A negative sign is present in the exponents to the **right** of the base (10^0).

Explanation of Exponents

⚠ The exponent 12 in 10^{12} signifies that the number 10^{12} has 12 zeros following the 1.

- In expanded form, $10^{12} = 1{,}000{,}000{,}000{,}000$. This number is read as "1 trillion."
- The chart below outlines various quantities, and shows how the exponent to which the base 10 is raised relates to their SI prefixes and SI symbols.

Name of Number	Base 10 and Corresponding Exponent	Expanded Form of Number	SI prefix	SI symbol
ten	10^1	10	deca	da
hundred	10^2	100	hecto	h
thousand	10^3	1,000	kilo	k
million	10^6	1,000,000	mega	M
billion	10^9	1,000,000,000	giga	G
trillion	10^{12}	1,000,000,000,000	tera	T

🔜 **Place Value** will be covered in a forthcoming section of this manual.

The SI Base Units

? What are the seven "SI base units"?

→ The **SI base units** are seven units of measure defined by the International System of Units as the *basic set* from which all other SI units can be derived.

The units and their physical quantities are listed in the chart below.

Length	meter (m)
Time	second (s)
Amount of Substance	mole (mol)
Electric Current	ampere (A)
Temperature	kelvin (K)
Luminous Intensity	candela (cd)
Mass	kilogram (kg)

👤 The SI base units form a set of mutually independent dimensions, which are commonly seen in "dimensional analysis" in science and technology courses.

🔜 **Dimensional Analysis** will be covered in a later section of this manual.

Combining Prefixes

⚠ Prefixes may **not** be used in combination.

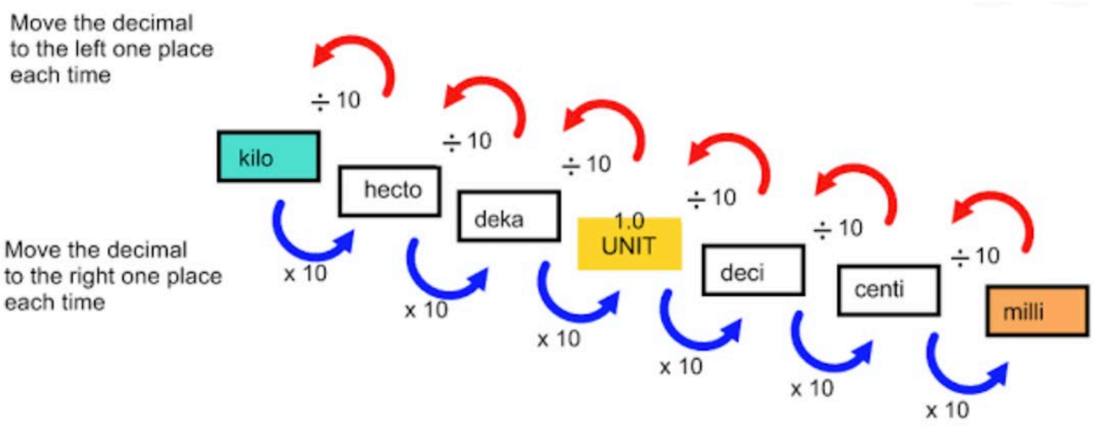
- This also applies to mass, for which the SI base unit $(kilogram)$ already contains a prefix.

For instance, milligram (mg) is used instead of $micro$$kilo$gram ($\mu kg$).

Conversion of Metric Units Using the "Metric Staircase"

How can the "**Metric Staircase**" be used to help students convert between metric units?

To convert between metric units, encourage students to draw the "**Metric Staircase**," pictured below.



Encourage students to use the "**Metric Staircase**" to convert the given quantities:

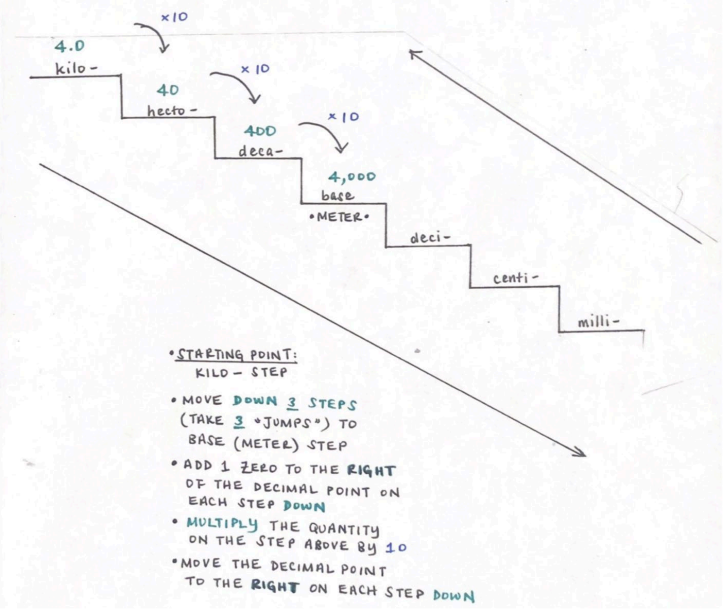
(a) $4\ km = \underline{\ \ ?\ \ }\ m$

(b) $9\ mg = \underline{\ \ ?\ \ }\ kg$

Answers:

(a) How many m are there in 4 km?

In this case, the base step is "**meter** $(\boldsymbol{m})$**,**" which is the **SI base unit** for **length**.



• STARTING POINT:
KILO – STEP

• MOVE DOWN 3 STEPS (TAKE 3 "JUMPS") TO BASE (METER) STEP

• ADD 1 ZERO TO THE RIGHT OF THE DECIMAL POINT ON EACH STEP DOWN

• MULTIPLY THE QUANTITY ON THE STEP ABOVE BY 10

• MOVE THE DECIMAL POINT TO THE RIGHT ON EACH STEP DOWN

(1) **Determine the starting point and the location of the given quantity's decimal point**.

- The starting point is the kilometer (km) step (km corresponds to the *kilo* step), as seen in the "**Metric Staircase**" above.

Because we do not see a decimal point in 4 km, it is located the **right** of the last digit!
We can add a zero to the right of the decimal point without changing the numerical value of the given quantity.

- Therefore, $4\ km = 4.0\ km$

(2) **Determine the final location (or "destination").**
Then, count the number of "jumps" to the destination.

- The destination is the **meter** $(\boldsymbol{m})$ step, which corresponds to the **base** step.
- We must make **3** "jumps" to get from the *kilometer* (km) step to the **meter** $(\boldsymbol{m})$ step.

(3) **Determine the direction of the "jump(s)," and move the decimal point the same number of places as the number of "jumps" made in Step (2) above, following the guideline below**:

- To get from the *kilometer* (km) step to the **meter** $(\boldsymbol{m})$ step, we must make 3 "jumps" **down**.
- We move the decimal point one place to the **right** with each step that we go **down.**
- We **add a zero to the right of the decimal point** with each step that we go **down**.

    - When we go from the kilometer (km) step **down** to the hectometer (hm) step, we move the decimal point in 4.0 one place to the **right** to obtain 40.
    - We add a zero to the **right** of the decimal point in 4.0 to obtain 40.

    - When we go from the hectometer (hm) step **down** to the decameter (dam) step, we move the decimal point in 40 one place to the **right** to obtain 400.
    - We add a zero to the **right** of the decimal point in 40 to obtain 400.

    - When we go from the decameter (dam) step **down** to the meter (m) step, we move the decimal point in 400 one place to the **right** to obtain 4,000.
    - We add a zero to the **right** of the decimal point in 400 to obtain 4,000.

(4) The numerical quantity when taking "jumps" up or down the "Metric Staircase" is determined by the operation (multiplication or division) which we perform on the quantity with each step up or down.

- We **multiply** the quantity on the step above by **10** with each step that we go **down**.
- This signifies that the given quantity will get **larger** with each step that we go **down**.

- As we go from kilometer (km) step **down** to the hectometer (hm) step, we **multiply** the quantity on the step above by **10**.

$$4.0\ km \bullet \mathbf{10} = 40\ hm$$

- As we go from the hectometer (hm) step **down** to the decameter (dam) step, we **multiply** the quantity on the step above by **10**.

$$40\ hm \bullet \mathbf{10} = 400\ dam$$

- As we go from the decameter (dam) step **down** to the meter (m) step, we **multiply** the quantity on the step above by **10**.

$$400\ dam \bullet \mathbf{10} = 4{,}000\ m$$

- Therefore, $4\ km = \mathbf{4,000}\ \boldsymbol{m}$.

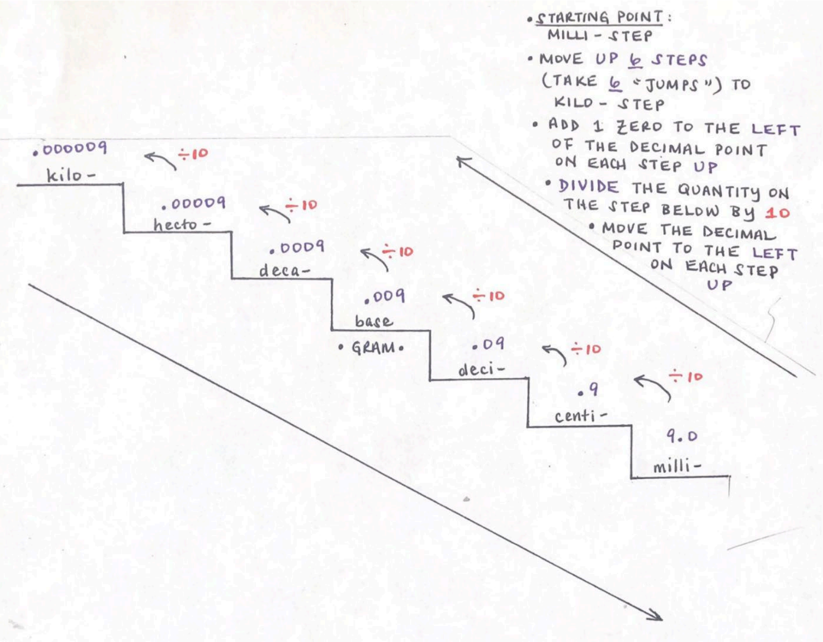
In other words, **3** "jumps" **down** is analogous to **multiplying** the original quantity (in this case, 4) by 10^3.

In a forthcoming section of this manual, we will learn to express $\mathbf{4,000}$ in **scientific notation** as $\mathbf{4.0 \bullet 10^3}$.

(b) $9\ mg =$ ___?___ kg

How many kg are there in 9 mg?

In this case, the base step is "**gram** $(\boldsymbol{g})$," which is the **SI base unit** for **mass**.



(1) **Determine the starting point and the location of the given quantity's decimal point**.

- The starting point is the milligram (mg) step (mg corresponds to the milli step), as seen in the "**Metric Staircase**" above.

  Because we do not see a decimal point in 9 mg, it is located the right of the last digit!
  We can add a zero to the right of the decimal point without changing the numerical value of the given quantity.

- Therefore, $9\ mg = \mathbf{9.0}\ \boldsymbol{mg}$

(2) **Determine the final location (or "destination").**
Then, count the number of "jumps" to the destination.

- The destination is the kilogram (kg) step.
- We must make **6** "jumps" to get from the milligram (mg) step to the kilogram (kg) step.

(3) **Determine the direction of the "jump(s)," and move the decimal point the same number of places as the number of "jumps" made in Step (2) above, following the guideline below**:

- To get from the milligram step to the kilogram step, we must make **6** "jumps" **up**.
- We move the decimal point one place to the **left** with each step that we go **up.**

- We **add a zero to the left of the decimal point** with each step that we go **up**.

  - When we go from the milligram (mg) step **up** to the centigram (cg) step, we move the decimal point in 9.0 one place to the **left** to obtain **.9**.
  - We add a zero to the **left** of the decimal point in 9.0 to obtain **.9.**

  - When we go from the centigram (cg) step **up** to the decigram (dg) step, we move the decimal point in .9 one place to the **left** to obtain **.09**.
  - We add a zero to the **left** of the decimal point in .9 to obtain **.09.**

  - When we go from the decigram (dg) step **up** to the gram (g) step, we move the decimal point in .09 one place to the **left** to obtain **.009**.
  - We add a zero to the **left** of the decimal point in .09 to obtain **.009.**

  - When we go from the gram (g) step **up** to the decagram (dag) step, we move the decimal point in .009 one place to the **left** to obtain **.0009**.
  - We add a zero to the **left** of the decimal point in .009 to obtain **.0009.**

  - When we go from the decagram (dag) step **up** to the hectogram (hg) step, we move the decimal point in .0009 one place to the **left** to obtain **.00009**.
  - We add a zero to the **left** of the decimal point in .0009 to obtain **.00009.**

  - When we go from the hectogram (hg) step **up** to the kilogram (kg) step, we move the decimal point in **.00009** one place to the **left** to obtain **.000009**.
  - We add a zero to the **left** of the decimal point in .00009 to obtain **.000009.**

(4) **The numerical quantity when taking "jumps" up or down the "Metric Staircase" is determined by the operation (multiplication or division) which we perform on the quantity with each step up or down.**

- We **divide** the quantity on the step below by **10** with each step that we go **up**.
- This signifies that the given quantity will get **smaller** with each step that we go **up**.

- As we go from the milligram (mg) step **up** to the centigram (cg) step, we **divide** the quantity on the step above by **10**.

$$9.0\ mg \div \mathbf{10} = .9\ cg$$

- As we go from the centigram (cg) step **up** to the decigram (dg) step, we **divide** the quantity on the step above by **10**.

$$.9\ cg \div \mathbf{10} = .09\ dg$$

- As we go from the decigram (dg) step **up** to the gram (g) step, we **divide** the quantity on the step above by **10**.

$$.09\ dg \div \mathbf{10} = \mathbf{.009\ g}$$

- As we go from the gram (g) step **up** to the decagram (dag) step, we **divide** the quantity on the step above by **10**.

$$.009\ g \div \mathbf{10} = .0009\ dag$$

- As we go from the decagram (dag) step **up** to the hectogram (hg) step, we **divide** the quantity on the step above by **10**.

$$.0009\ dag \div \mathbf{10} = .00009\ hg$$

- As we go from the hectogram (hg) step **up** to the kilogram (kg) step, we **divide** the quantity on the step above by **10**.

$$.00009\ hg \div \mathbf{10} = \mathbf{.000009\ kg}$$

💡Therefore, $9\ mg = \mathbf{.000009\ kg}$

(c) In other words, **6** "jumps" **up** is analogous to **dividing** the original quantity (in this case, 9) by $10^{\mathbf{6}}$.

🔜 In a forthcoming section of this manual, we will learn to express $\mathbf{.000009}$ in **scientific notation** as $\mathbf{9.0 \bullet 10^{-6}}$.

Expressing Numbers in Standard Form in Scientific Notation

What is "scientific notation"?

→ **Scientific notation** is a standard way of writing a very small number or a very large number in a compact form.

- Numbers written in scientific notation are easier to use in computations.

How do we write a number in scientific notation?

→ To write a number in scientific notation, we follow the form:

$$c \bullet 10^{a},$$

- where c is a number between 1 and 10, but not 10. a is an integer

Recall: An integer is a positive or negative number.

⚠ The base is always 10!

The Scientific Notation Game

The following is a kinesthetic aid designed as a "game" to teach students to write numbers in **scientific notation**.

The steps of the "game" are outlined below:

Step 1: **Identify the location of the original decimal point**.

- Peer mentors may wish to highlight the original decimal point in one color.

Step 2: **Identify the final location (or "destination") of the original decimal point**.

- The goal is to move (or "jump") the original decimal point to the **left** or the **right** to create a number from 1 up to 10, but not including 10.

Step 3: **Move the original decimal point to its final location to arrive at c, a number between 1 and 10, but not 10**.

Step 4: **Determine a, the exponent to which the base 10 is raised.**

- **The value of the exponent is the number of "jumps" of the original decimal point.**
- **Then, determine the sign of the exponent.**

- When the original decimal point is moved to the **left**, the exponent that the base 10 is raised to is **positive**.
- When the original decimal point is moved to the **right**, the exponent that the base 10 is raised to is **negative**.

Step 5: **Using the above rules, write the number in the form:** $c \bullet 10^{a}$.

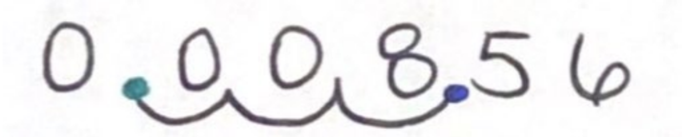
Encourage students to express the following in scientific notation:

(a) 0.00856

(b) 713,000

Answers:

(a) 0.00856 is a small number which can be written in scientific notation as follows:



Step 1: **Identify the location of the original decimal point.**

The original decimal point is in **green**.

Step 2: **Identify the final location (or "destination") of the original decimal point.**

The "destination" for the original decimal point is in **blue**.

Step 3: **Move the original decimal point to its final location to arrive at c, a number between 1 and 10 but not 10.**

When the decimal point is moved, we arrive at $c = 8.56$

Step 4: **Determine the exponent to which the base 10 is raised.**

- **The value of the exponent is the number of "jumps" of the original decimal point.**
- **Then, determine the sign of the exponent.**

When moving from 0.00856 to 8.56, we made **3** "jumps" to the **right**.

- When the original decimal point is moved to the **right**, the exponent that the base 10 is raised to is **negative**.

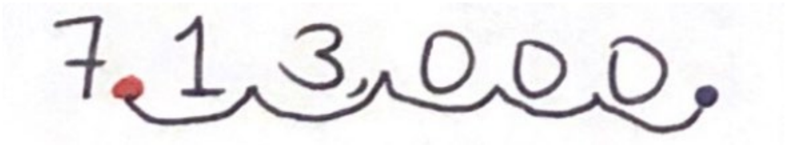
Therefore, c is multiplied by $\mathbf{10^{-3}}$.

Step 5: **Using the above rules, write the number in the form:** $c \bullet 10^a$.

$$c = 8.56$$
$$10^a = 10^{-3}$$

Expressed in **scientific notation**, 0.00856= $\mathbf{8.56 \bullet 10^{-3}}$

(b) 713,000 is a large number which can be written in scientific notation as follows:



Because we do not see a decimal point in 713,000 , it is located the **right** of the last digit (in this case, 0)!

Step 1: **Identify the location of the original decimal point.**

The original decimal point is in **red**.

Step 2: **Identify the final location (or "destination") of the original decimal point.**
The "destination" for the original decimal point is in **purple**.

Step 3: **Move the original decimal point to its final location to arrive at c, a number between 1 and 10 but not 10.**
When the decimal point is moved, we arrive at $c = 7.13$

Step 4: **Determine the exponent to which the base 10 is raised.**
•The value of the exponent is the number of "jumps" of the original decimal point.
•Then, determine the sign of the exponent.

When moving from 713,000 to 7.13, we made 5 "jumps" to the **left**.
•When the original decimal point is moved to the **left**, the exponent that the base 10 is raised to is **positive**.
Therefore, c is multiplied by 10^5.

Step 5: **Using the above rules, write the number in the form:** $c \bullet 10^a$.

$$c = 7.13$$
$$10^a = 10^5$$

Expressed in **scientific notation**, 713,000= $7.13 \bullet 10^5$

Expressing Numbers in Scientific Notation in Standard Form

How do we express numbers in standard form that are written in scientific notation?

The "**Scientific Notation Game**" can be used as a tool to work ***backwards***, when given a number in **scientific notation** which is to be expressed in **standard form**.

The terms "standard form" and "expanded form" can be used interchangeably.

Encourage students to express the following **in standard form**:
(a) $5.8 \bullet 10^8$
(b) $8.72 \bullet 10^{-4}$

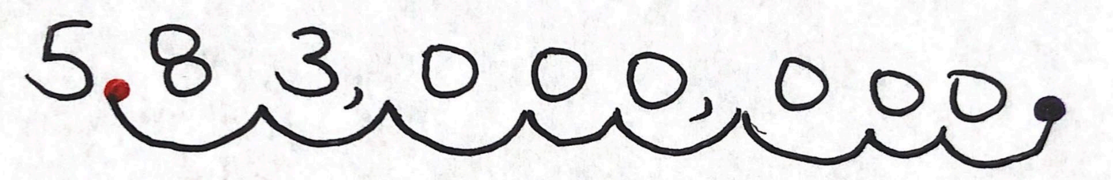
(a) $5.83 \bullet 10^8$

• Because the exponent to which the base 10 is raised is **positive**, we know we are looking for a **large** number.

• In order to make 5.83 larger, we must move the decimal point to the **right**.

- The original decimal point is in **red**.
- The “destination” for the original decimal point is in **purple**.

- The exponent to which the base 10 is raised is **8**.
- Therefore, we must move the original decimal point 8 “jumps” to the **right**.
- We fill in the loops with zeros, as follows:



Therefore, we write $5.83 \bullet 10^{8}$ in standard form as: $583,000,000$

(b) $8.71 \bullet 10^{-4}$

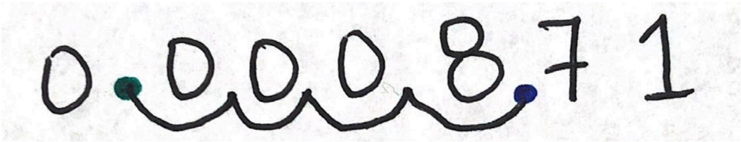
- Because the exponent to which the base 10 is raised is **negative**, we know we are looking for a **small** number.

- In order to make 8.71 smaller, we must move the decimal point to the **left**.

- The original decimal point is in **blue**.
- The “destination” for the original decimal point is in **green**.

- The exponent to which the base 10 is raised is **4**.
- Therefore, we must move the original decimal point 4 “jumps” to the **left**.

- We fill in the loops with zeros, as follows:



Therefore, we write $8.71 \bullet 10^{-4}$ in standard form as: 0.000871

Order of Operations

⚠ A point of confusion for students may occur when they see more than one operation (addition, subtraction, multiplication, or division) in the same expression.

In what **order are the four operations performed** if more than one operation appears in the same expression?

To eliminate any ambiguity, mathematicians have agreed that the proper order of operations is: **P**arentheses, **E**xponents, **M**ultiplication *and* **D**ivision, **A**ddition *and* **S**ubtraction.

Although multiplication is listed before division, these operations are performed **left to right** in order of appearance.

- Similarly, addition and subtraction are performed **left to right** in order of appearance.

The mnemonic device **PEMDAS**: **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally is often used to remember this order.

Encourage students to use the proper **order of operations** to simplify the following expressions:

(a) $5^3 - 4(1 + 2)^2$

(b) $11 - 4 \div 2 \bullet 5 + 3$

Answers:

(a) $5^3 - 4(1 + 2)^2 = 5^3 - 4 \bullet 3^2$

$= 125 - 4 \bullet 9$

$= 125 - 36$

$= \mathbf{89}$

- Note: The colors above correspond to the operations performed in the correct order, using the mnemonic device **PEMDAS**.

⚠ If we do not see an operation symbol in between a number and a parenthesis, it is implied that the operation is multiplication.

- For instance, in the example above, $4(1 + 2)^2$ means $4 \bullet (1 + 2)^2$

(b) $11 - 4 \div 2 \bullet 5 + 3 = 11 - 2 \bullet 5 + 3$

$= 11 - 10 + 3$

$= 1 + 3$

$= \mathbf{4}$

  - Note: The colors above correspond to the operations performed in the correct order, using the mnemonic device **PEMDAS**.

🧠💭 Addition and subtraction are performed **left to right** in order of appearance.

  - For instance, in the example above, both addition and subtraction appear in the expression.
  - Subtraction is performed first because it appears to the left of addition.
  - Then, addition is performed.

Performing Operations with Signed Numbers

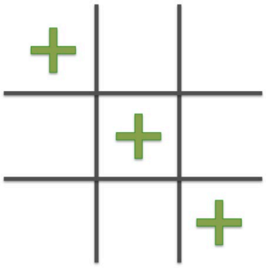
⚠ Students often have trouble performing addition, subtraction, multiplication, and division of signed numbers.

Multiplication and Division of Signed Numbers

What is a best practice pedagogical strategy to teach multiplication and division of signed numbers?

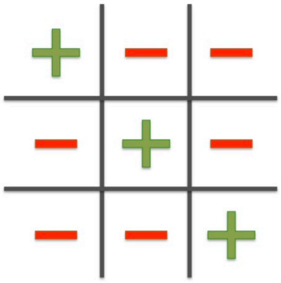
Peer mentors can encourage students to use a **Tic Tac Toe** board to teach students to multiply and divide signed numbers.

- Step 1: Peer mentors insert positive (+) signs on the diagonal *(from upper left to lower right)* of the **Tic Tac Toe** board, as shown below:



+		
	+	
		+

- Step 2: Peer mentors ask students to insert negative (−) signs in all remaining spaces on the **Tic Tac Toe** board, as shown below:



+	−	−
−	+	−
−	−	+

  - *Row 1*: A positive (+) number multiplied (or divided) by a negative (−) number equals a negative (−) number.
  - *Row 2*: A negative (−) number multiplied (or divided) by a positive (+) number equals a negative (−) number.
  - *Row 3*: A negative (−) number multiplied (or divided) by a negative (−) number equals a positive (+) number.
  - *Diagonal (from upper left to lower right)*: A positive (+) number multiplied (or divided) by a positive (+) number equals a positive (+) number.

When solving problems involving multiplication and division of signed numbers, peer mentors can ask students the following questions:

(1) **What is the sign** (+ or −) **of the product or quotient?**

(2) **What is the numerical value of the product or quotient?**

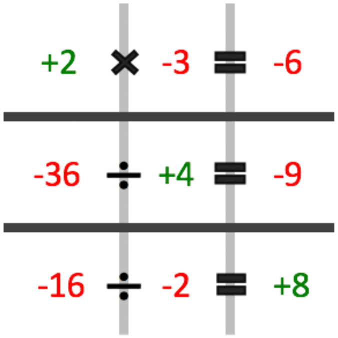
A product is the result which is obtained by multiplying one quantity by another.

A quotient is the result which is obtained by dividing one quantity by another.

Multiplication can be represented by an **x** or a • symbol.

What are some applications involving multiplication and division of signed numbers?

Encourage students to find the product or quotient, using the **Tic Tac Toe** board below:



+2	×	-3	=	-6
-36	÷	+4	=	-9
-16	÷	-2	=	+8

- *Row 1*: $+2 \times -3 = -6$

(1) **What is the sign of the product or quotient?**

- A positive (+) number multiplied by a negative (−) number equals a negative (−) number.

Therefore, the sign of the product is negative (−).

(2) **What is the numerical value of the product or quotient?**

- $2 \times 3 = 6$

Therefore, $+2 \times -3 = \mathbf{-6}$

- *Row 2*: $-36 \div +4 = -9$

(1) **What is the sign of the product or quotient?**

- A negative (−) number divided by a positive (+) number equals a negative (−) number.

Therefore, the sign of the quotient is negative (−).

(2) **What is the numerical value of the product or quotient?**

- $36 \div 4 = 9$

Therefore, $-36 \div +4 = \mathbf{-9}$

- Row 3: $-16 \div -2 = +8$

(1) **What is the sign of the product or quotient?**

- A negative ($-$) number divided by a negative ($-$) number equals a positive ($+$) number.

Therefore, the sign of the quotient is positive ($+$).

(2) **What is the numerical value of the product or quotient?**

- $16 \div 2 = 8$

Therefore, $-16 \div -2 = +8$

- *Diagonal (from upper left to lower right)*: $+2 \times +4 = +8$

(1) **What is the sign of the product or quotient?**

- A positive ($+$) number multiplied by a positive ($+$) number equals a
- positive ($+$) number.

Therefore, the sign of the product is positive ($+$).

(2) **What is the numerical value of the product or quotient?**

- $2 \times 4 = 8$

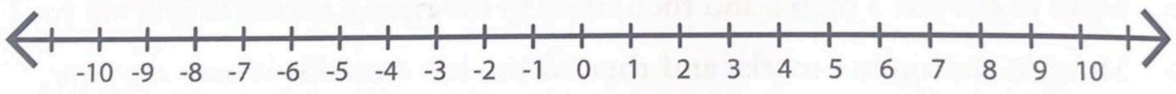
Therefore, $+2 \times +4 = +\mathbf{8}$

Addition and Subtraction of Signed Numbers

What is a best practice pedagogical strategy to teach addition and subtraction of signed numbers?

Peer mentors can encourage students to draw a **number line** to represent positive ($+$) and negative ($-$) numbers.

- A **number line** has a zero at the center, with positive numbers moving off to the right, and negative numbers moving off to the left, as shown below:

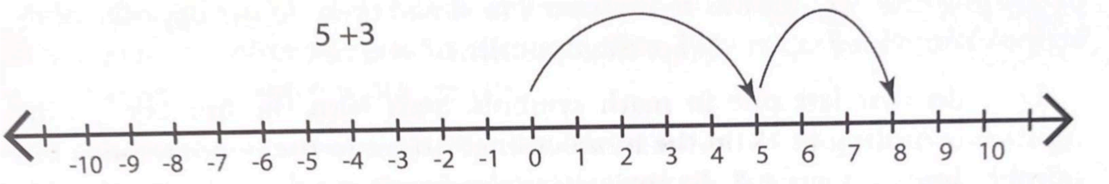


What are some applications involving addition and subtraction of signed numbers?

A sum is the result which is obtained by adding one quantity to another.
A difference is the result which is obtained by subtracting one quantity from another.

Encourage students to evaluate the following, using a **number line**.

(a) 5 + 3



- Begin by moving five places to the **right** of the zero on the **number line**.
- To add 3 to 5, we move three more places to the **right**.
- The sum is eight places to the **right**, which is equivalent to +8.

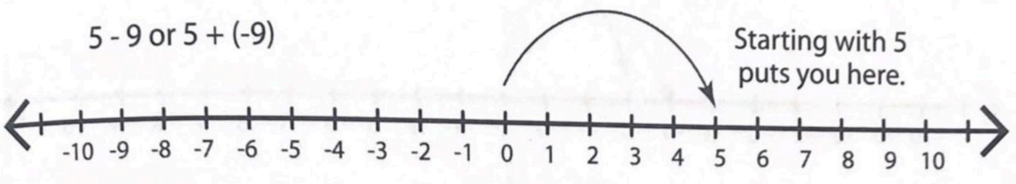
Therefore, 5 + 3 = **8**

If there is not a plus (+) or minus (−) sign in front of a number, the number is assumed to be positive (+).

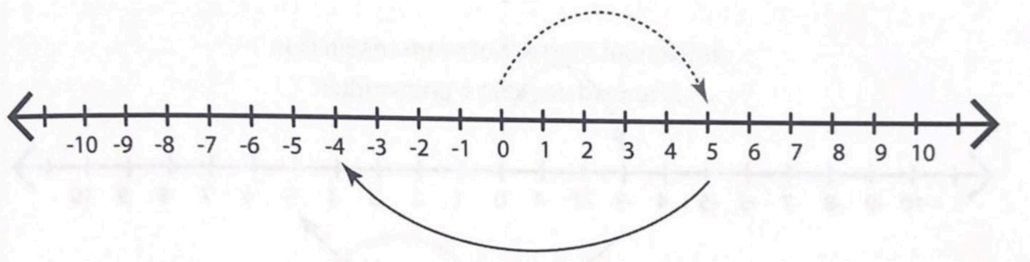
Subtraction is the Same as Adding the Opposite

(b) Encourage students to evaluate 5 − 9, using a **number line**:

Recall: 5 − 9 = 5 + (−9)



- Begin by moving five places to the **right** of the zero on the **number line**.
- Subtracting 9 means moving to the **left** nine places.
  Adding a (−9) also means moving to the **left** nine places.



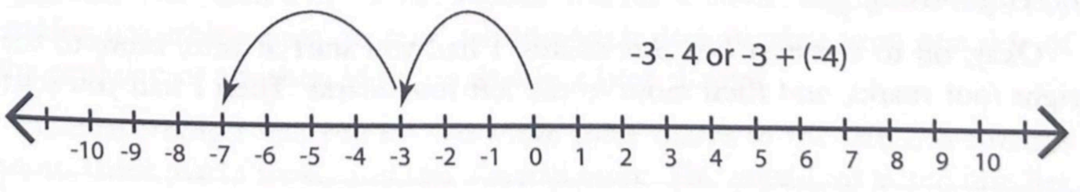
  - The sum is four places to the **left**, which is equivalent to −4.

Therefore, 5 − 9 = 5 + (−9) = **−4**

Adding and Subtracting Numbers
When One or Both are Negative

How do we add and subtract numbers when one or both are negative ($-$)?

(c) Encourage students to evaluate $-3-4$, using a **number line**:



Recall: $-3-4=-3+(-4)$

- Begin by moving three places to the **left** of the zero on the **number line**.
- Subtracting 4 means moving to the **left** four places.
  Adding a (-4) also means moving to the **left** nine places.

- The sum is seven places to the **left**, which is equivalent to -7.

Therefore, $-3-4=-3+(-4)=\mathbf{-7}$

The Additive Inverse

What is meant by the "additive inverse" of a number?

→ The **additive inverse** of number c, is the number that, when added to c, yields zero.

- Therefore,

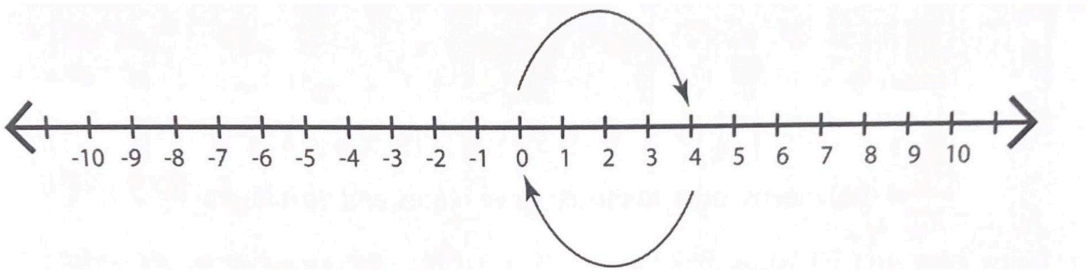
$$(\boldsymbol{a\ number})+(\boldsymbol{its\ additive\ inverse})=\mathbf{0}$$

This number is sometimes referred to as the "opposite [number]," or the "negative" of a number.

How do we represent the **additive inverse** on a **number line**?

(a) Encourage students to evaluate $4-4$, using a **number line**, and to find the **additive inverse** of 4.

Recall: $4-4=4+(-4)$

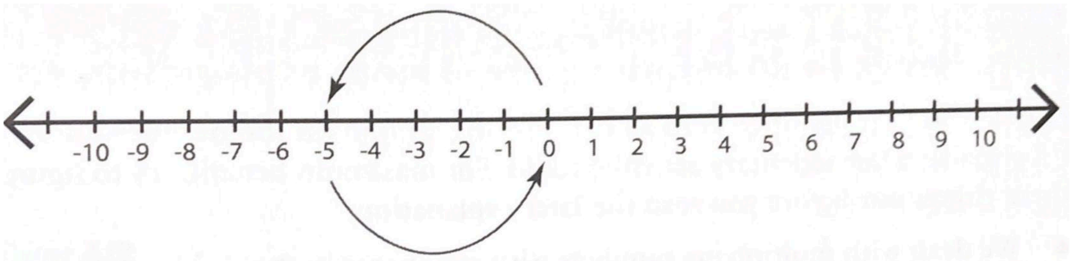


- Begin by moving four places to the right of the zero on the **number line**.
- Subtracting 4 means moving to the left four places.
- The sum is back at 0.

💡Therefore, $4 - 4 = 4 + (-4) = 0$

• The **additive inverse** of 4 is: **(-4)**.

(b) Encourage students to evaluate $-5 + 5$, using a **number line**, and to find the **additive inverse** of -5.



- Begin by moving five places to the left of the zero on the **number line**.
- Adding 5 means moving to the right five places.
- The sum is back at 0.

💡Therefore, $-5 + 5 = 0$

- The **additive inverse** of -5 is: **5**.

The **multiplicative inverse** will be covered in a later section of this manual.

Reciprocals

⚠ A point of confusion for students may occur when they are asked to take the **reciprocal** of a number.

What is meant by the “reciprocal of a number”?

→ **The reciprocal of a number is 1 divided by the number**.

- Another name for the reciprocal of a number is its “multiplicative inverse.”
- **The reciprocal of a fraction is found by flipping its numerator and denominator**.

Any whole number can be written as a fraction whose numerator is the whole number and whose denominator is 1.

(i.e. -3 can be written as $\frac{-3}{1}$)

○ The **multiplicative inverse** of $\frac{-3}{1}$ is: $\mathbf{\frac{-1}{3}}$

$\frac{-3}{1} \bullet \frac{-1}{3} = 1$

AB Algebraically,

$$(\boldsymbol{a\ number}) \bullet (\boldsymbol{its\ reciprocal}) = \mathbf{1}$$

The Multiplicative Inverse

What is meant by the “multiplicative inverse” of a number?

→ The **multiplicative inverse** of a number d, is the number that, when multiplied by d, yields 1.

- Therefore, $(\boldsymbol{a\ number}) \bullet (\boldsymbol{its\ multiplicative\ inverse}) = \mathbf{1}$

Encourage students to find the **multiplicative inverse** of $\frac{15}{6}$.

Ask students what number can be multiplied by $\frac{15}{6}$ to yield 1?

- The **reciprocal** of $\frac{15}{6}$ is $\mathbf{\frac{6}{15}}$.

$$\left(\frac{\overset{\boxed{1}}{\cancel{15}}}{\underset{\boxed{1}}{\cancel{6}}}\right) \bullet \left(\frac{\overset{\boxed{1}}{\cancel{6}}}{\underset{\boxed{1}}{\cancel{15}}}\right) = \mathbf{1}$$

Numerator • Numerator = 1 • 1 = 1

Denominator • Denominator = 1 • 1 =

Therefore, the **multiplicative inverse** of $\frac{15}{6}$ is: $\mathbf{\frac{6}{15}}$.

0 does **not** have a **multiplicative inverse** because no real number multiplied by 0 yields 1.

The Negative Reciprocal of a Fraction

⚠ Students often think that ***negative*** **reciprocals** are negative ($-$) in sign.
  - Peer mentors should dispel this misconception by using the chart below.

❓Numerically, what is meant by the "***negative*** **reciprocal**" of a fraction?

Fraction	**Reciprocal of Fraction**	**Negative Reciprocal of Fraction**
$\frac{1}{2}$	$\frac{2}{1}$	$-\frac{2}{1}$
$-\frac{3}{4}$	$-\frac{4}{3}$	$\frac{4}{3}$
$\frac{0}{9}$	$\frac{9}{0}$	$-\frac{9}{0} = \text{undefined}$

  - In *Row 2*, we see that the negative reciprocal $(\frac{4}{3})$ of the fraction $(-\frac{3}{4})$ is **not** negative ($-$) in sign!

Fractions with Numerators = 0 or Denominators = 0

⚠ A point of confusion for students may occur when they see fractions involving $numerators = 0$ or $denominators = 0$.

  - In *Row 3*, we see that the fraction $\frac{0}{9}$ has negative reciprocal $-\frac{9}{0}$.
  - $\frac{0}{9} = 0$

    🧠💭 $\frac{0}{any\ quantity} = 0$

  - $-\frac{9}{0}$ is undefined.

🧠💭 $\frac{any\ quantity}{0}$ is undefined.

  - Therefore, the reciprocal of the fraction $\frac{0}{9} = 0$ is undefined.

Division by 0

⚠ A point of confusion for students may occur when they see 0 in the denominator of a fraction.

It is important to remind students that $\frac{\boldsymbol{any\ quantity}}{\mathbf{0}}$ is **undefined**.

Why is $\frac{\boldsymbol{any\ quantity}}{\mathbf{0}}$ **undefined**?

Peer mentors may ask the following questions to help students discover that $\frac{\boldsymbol{any\ quantity}}{\mathbf{0}}$ is **undefined**.

- Question 1:

What is the value of $\frac{10}{5}$?

College students know that $\frac{10}{5} = 2$.

However, it is important to remind students that in order to "check" this **quotient**, we use the **inverse operation** of division, which is multiplication, as follows.

A quotient is the result which is obtained by dividing one quantity by another.

An **inverse operation** "undoes" what was done by the previous operation.

- The inverse operation of addition is subtraction, and vice versa.
- The inverse operation of multiplication is division, and vice versa.

- To check that $\frac{10}{5} = 2$, we use multiplication (the inverse operation of division), as follows:

$$quotient \bullet \ denominator = numerator.$$

☑ Check: $2 \bullet 5 = 10$

Therefore, $\frac{10}{5} = 2$.

- Question 2:

What is the value of $\frac{-64}{16}$?

College students know that $\frac{-64}{16} = -4$.

However, it is important to remind students that in order to “check” this division problem, we multiply, as follows:

$$quotient \bullet\ denominator = numerator.$$

☑ Check: $-4 \bullet 16 = -64$

Therefore, $\frac{-64}{16} = -4$.

- Question 3:

What is the value of $\frac{13}{0}$?

Instead of merely telling students that $\frac{13}{0}$ is undefined, peer mentors should encourage them to **list possible solutions**, and **use the guidelines from the first two examples above**, as follows:

- Potential Solution 1:
- Does $\frac{13}{0} = 0$?

☑ Check: $0 \bullet 0 \neq 13$

Therefore, $\frac{13}{0} \neq\ 0.$

- Potential Solution 2:
- Does $\frac{13}{0} = 13$?

☑ Check: $13 \bullet 0 \neq 13$

Therefore, $\frac{13}{0} \neq\ 13.$

- Potential Solution 3:
- Does $\frac{13}{0} = any\ quantity$?

☑ Check: $any\ quantity \bullet 0 \neq 13$

$any\ quantity \bullet 0 = 0$

Because **no** quantity exists whereby

$$quotient\ \bullet\ 0 = 13,$$

it follows that $\frac{\boldsymbol{any\ quantity}}{\mathbf{0}}$ is **undefined**.

“Cancel” vs. Division

⚠ Students often use the term “cancel” when simplifying fractions.

There are four operations (addition, subtraction, multiplication, and division).
- ○ Note: each of these operations end in **-ion**.

- On a calculator, the operation of **addition** is represented by a + symbol.

- On a calculator, the operation of **subtraction** is represented by a – symbol.

- On a calculator, the operation of **multiplication** is represented by a **x** symbol.
  Recall: **Multiplication** can also be represented by a • symbol.

- On a calculator, the operation of **division** is represented by a ÷ symbol.
  Recall: **Division** can also be represented by a / symbol.

A symbol for **CANCEL** does **not** exist on a calculator!

⚠ “Cancel” does **not** mean “cross out,” or “delete.”

☑ When students say “cancel,” it is important for peer mentors to inform them that they are, in fact, dividing out common factors from both the numerator and the denominator of a fraction.

- Therefore, the word “cancel” should be replaced by “division.”

Simplifying Fractions by Dividing Out the GCF

When simplifying fractions, encourage students to use the following guidelines:

(1) **Factor the numerator and denominator completely**.
Remind students to factor out the **Greatest Common Factor** (**GCF**).
- The **Greatest Common Factor** (**GCF**) is represented in gray in the examples below.

(2) Divide the numerator and the denominator by the **GCF**.

Encourage students to simplify the following:

(a) $\frac{36}{12}$

(b) $\frac{-30}{45}$

(c) $\frac{x^2-1}{x^2+x-2}$

(a) $\frac{36}{12} = \frac{12 \bullet 3}{12} = \frac{3}{1} = 3$

$$\frac{36}{12} = \frac{\overset{\boxed{1}}{\cancel{12}} \bullet 3}{\underset{\boxed{1}}{\cancel{12}}} = \frac{3}{1} = \mathbf{3}$$

$$\boxed{GCF = 12} \qquad \boxed{\frac{12}{12} = 1}$$

(b) $\frac{-30}{45} = \frac{-15 \bullet 2}{15 \bullet 3} = -\frac{2}{3}$

$$\frac{-30}{45} = \frac{\overset{\boxed{1}}{\cancel{-15}} \bullet 2}{\underset{\boxed{1}}{\cancel{15}} \bullet 3} = -\frac{2}{3}$$

$$\boxed{GCF = 15} \qquad \boxed{\frac{15}{15} = 1}$$

Recall: From the **"Tic Tac Toe" board used to** multiply **and** divide **signed numbers,** a negative quantity ($-$) divided by a positive quantity ($+$) is a negative ($-$) quantity.

(c) $\frac{x^2-1}{x^2+x-2} = \frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$

$$\frac{x^2-1}{x^2+x-2} = \frac{\overset{\boxed{1}}{\cancel{(x-1)}}(x+1)}{\underset{\boxed{1}}{\cancel{(x-1)}}(x+2)} = \frac{x+1}{x+2}$$

$$\boxed{GCF = x-1} \qquad \boxed{\frac{x-1}{x-1} = 1}$$

As an aside, it is important to remind students that they **cannot** divide out x in the numerator and denominator of the fraction $\frac{x-1}{x-1}$ **.**

⚠While it appears that x is common to both the numerator and denominator of the fraction $\frac{x-1}{x-1}$, it is **not** a common factor of both the numerator and denominator of the fraction $\frac{x-1}{x-1}$.

- Therefore, x **cannot** be divided out.
- The fraction $\frac{x-1}{x-1}$ is in simplest form.

Simplifying Complex Fractions

⚠ A point of confusion for students may occur when they must simplify "**complex fractions**."

- A **complex fraction** is a fraction in which the numerator, the denominator, or both, are themselves fractional expressions.

To simplify **complex fractions**, peer mentors can encourage students to visit the restaurant "**KCF**."

- "**KCF**" is different from the popular restaurant chain "**KFC**" (**K**entucky **F**ried **C**hicken).

KCF – stands for **K**eep **C**hange **F**lip!

- **K**eep means **K**eep the numerator as is.
- **C**hange means **C**hange the division of the numerator by the denominator to multiplication.
- **F**lip means **F**lip the denominator.

Recall: **F**lipping a fraction is analogous to taking its **reciprocal**!

Encourage students to simplify the following **complex fraction** using **KCF** and by factoring out the GCF:

$$\frac{-\frac{1}{2}}{\frac{3}{4}}$$

Answer:

- **K**eep the numerator as is: $-\frac{1}{2}$
- **C**hange means **C**hange the division of the numerator by the denominator to multiplication.
- **F**lip means **F**lip the denominator.

- Simplify by using **KCF** and by factoring out the **GCF**, as follows:

$$-\frac{1}{\underset{\boxed{1}}{\cancel{2}}} \bullet \frac{\overset{\boxed{1}}{\cancel{2}}(2)}{3} = -\frac{2}{3}$$

$$\boxed{\textbf{Numerator} \bullet \textbf{Numerator} = -1 \bullet 2 = -2}$$

$$\boxed{\textbf{Denominator} \bullet \textbf{Denominator} = 1 \bullet 3 = 3}$$

Common Algebraic Misconceptions Involving Exponents
(…and How to Dispel Them!)

- Common Student Misconception 1:

⚠ A point of confusion for students may occur when they are asked to **multiply powers of the same base**.

☑ To **multiply powers of the same base**, **keep the base**, and **add the exponents**:

$$x^n x^m = x^{n+m}$$

Encourage students to simplify: $2^3 \bullet 2^8$

💡Answer: The bases are the same. **Keep the base**, and **add the exponents**.

🛑 Do **not** multiply the bases!

💡 Therefore, $2^3 \bullet 2^8 = 2^{3+8} = \mathbf{2^{11}}$

- Common Student Misconception 2:

⚠ A point of confusion for students may occur when they are asked to **divide powers of the same base**.

☑ To **divide powers of the same base**, **keep the base**, and **subtract the exponents**:

$$\frac{x^n}{x^m} = x^{n-m}$$

Encourage students to simplify: $\frac{6^9}{6^4}$

💡Answer: The bases are the same. **Keep the base**, and **subtract the exponents**.

🛑 Do **not** divide the bases!

💡 Therefore, $\frac{6^9}{6^4} = 6^{9-4} = \mathbf{6^5}$

- Common Student Misconception 3:

⚠ A point of confusion for students may occur when they are asked to **raise a power to a power**.

☑ To **raise a power to a power, multiply the exponents:**

$$(x^n)^m = x^{nm}$$

Encourage students to simplify: $(8^2)^7$

💡$(8^2)^7 = 8^{2 \bullet 7} = \mathbf{8^{14}}$

- Common Student Misconception 4:

⚠ A point of confusion for students may occur when they are asked to **distribute an exponent over multiplication**.

☑ An **exponent can be distributed over multiplication**:

$$(xy)^n = x^n y^n$$

Encourage students to simplify: $(5x)^2$

💡Answer: $(5x)^2 = 5^2 \bullet x^2 = \mathbf{25x^2}$

- Common Student Misconception 5:

⚠ A point of confusion for students may occur when they are asked to **distribute an exponent over division**.

☑ An **exponent can be distributed over division**:

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \qquad (y \neq 0)$$

Encourage students to simplify: $\left(\frac{s^3}{t^9}\right)^5$

💡Answer: $\left(\frac{s^3}{t^9}\right)^5 = \frac{(s^3)^5}{(t^9)^5} = \frac{\mathbf{s^{15}}}{\mathbf{t^{45}}}$

- Common Student Misconception 6:

⚠ A point of confusion for students may occur when they are asked to **distribute an exponent over addition or subtraction**.

☑ We do **not** distribute exponents over addition and subtraction:

$$(x + y)^n \neq x^n + y^n$$

$$(x - y)^n \neq x^n - y^n$$

  - For instance, $(x + y)^2 \neq x^2 + y^2$
  - Similarly, $(x - y)^2 \neq x^2 - y^2$

- Common Student Misconception 7:

⚠ A point of confusion for students may occur when they see **zero as an exponent**.

  - Students may expect that: b^0 ought to be equal to 0. This is **not** true.
  - We know that: $\frac{x^n}{x^m} = x^{n-m}$
  - $\frac{x^n}{x^n} = x^{n-n} = x^0$

- When we reduce the fraction, we see that : $\frac{x^n}{x^n} = 1$

☑ From this, we get the following rule:

- **Any nonzero number raised to the zero power equals 1**.

$$x^0 = 1 \qquad (x \neq 0)$$

🛑 0^0 is **undefined**.

Encourage students to note the following:

- $2^0 = 1$
- $(-158)^0 = 1$
- $\left(\frac{1}{3}\right)^0 = 1$
- $(x^2y^5z^{18})^0 = 1$

Encourage students to evaluate the following: $3x^0 - (2x)^0$ for $x \neq 0$

💡<u>Answer</u>:

Encourage students to pay careful attention to the base of each exponent.

- In $3x^0$, the exponent applies only to x (the base it is "touching"), not the 3.
  - Thus, $3x^0 = 3(1) = 3$
- Because of the parentheses in $(2x)^0$, the exponent applies to both the 2 and the x.
  - Thus, $(2x)^0 = 1$

💡Therefore, $3x^0 - (2x)^0 = 3 - 1 = \mathbf{2}$

- <u>Common Student Misconception 8</u>:

⚠ A point of confusion for students may occur when they see **negative exponents**.

☑ If x is any real number $(x \neq 0)$ and n is a positive integer, then:

$$\boldsymbol{a^{-n} = \frac{1}{a^n}}$$

Encourage students to evaluate the following: $(-2)^{-5}$

💡<u>Answer</u>: $(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-32} = -\frac{\mathbf{1}}{\mathbf{32}}$

- Common Student Misconception 9:

⚠ A point of confusion for students may occur when they see **a fraction raised to a negative power**.

☑ To **raise a fraction to a negative power, take the reciprocal of the fraction, and change the sign of the exponent**:

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

Encourage students to evaluate the following: $\left(\frac{s}{3t^4}\right)^{-2}$

💡Answer: $\left(\frac{s}{3t^4}\right)^{-2} = \left(\frac{3t^4}{s}\right)^{2} = \frac{3^2 \bullet t^8}{s^2} = \frac{9t^8}{s^2}$

- Common Student Misconception 10:

⚠ A point of confusion for students may occur when they see **negative exponents in numerators and denominators of fractions**.

☑ To **move a number raised to a power from numerator to denominator or from denominator to numerator, change the sign of the exponent**:

$$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

Encourage students to evaluate the following: $\frac{6st^{-4}}{2s^{-2}t^2}$

💡Answer: $\frac{6s \bullet s^2}{2t^2 \bullet t^4}$

s^{-2} moves to the numerator and becomes s^2

t^{-4} moves to the denominator and becomes t^4

$$\frac{\overset{\boxed{3}}{\cancel{6}}s \bullet s^2}{\underset{\boxed{1}}{\cancel{2}}t^2 \bullet t^4} = \frac{3s^3}{t^6}$$

Place Value

⚠ A point of confusion for students may occur when they are asked to **identify the place value of a digit in a number.**

? How do we identify the “place value” of a digit in a number?

→ Every digit in a number has a **place value**.

- The **place value** of each digit in a base-10 number **is determined by its position with respect to the decimal point**.
- **Each position represents multiplication by a power of 10**.

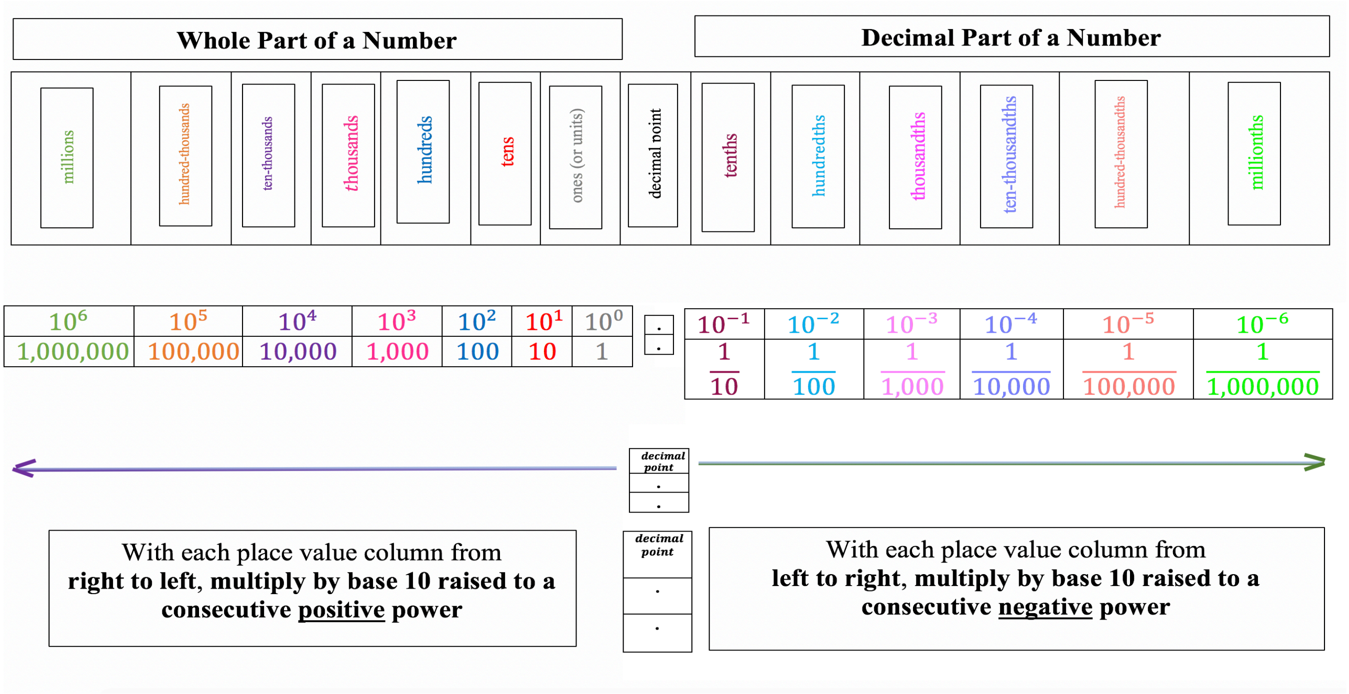
  🗣 As we move to the left and right of the decimal point on a place value chart, we **multiply the quantity by multiples of 10**.

- The place value of a digit **increases by multiples of 10** as we move to the **left** of the decimal point on a place value chart.
- The place value of a digit **decreases by multiples of 10** as we move to the **right** of the decimal point on a place value chart.

? What is a “place value chart”?

→ A **place value chart**, as seen below, can help students identify and compare the **place value** of the digits in a number.

Place Value Chart



Whole Part of a Number								**Decimal Part of a Number**					
millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones (or units)	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
10^6	10^5	10^4	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
1,000,000	100,000	10,000	1,000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1{,}000}$	$\frac{1}{10{,}000}$	$\frac{1}{100{,}000}$	$\frac{1}{1{,}000{,}000}$

With each place value column from **right to left, multiply by base 10 raised to a consecutive <u>positive</u> power**

With each place value column from **left to right, multiply by base 10 raised to a consecutive <u>negative</u> power**

The **decimal part** (or the **fractional part**) of a given numerical quantity is represented to the **right** of the decimal point.

- When moving to the **right** of the decimal point, we are moving one place value **smaller** with each consecutive column read **left to right of the decimal point**, as shown in the chart above.

It is important to tell students that when moving to the **right** of the decimal point, the given numerical quantity is to be **multiplied** by the corresponding multiple of 10.

If x is any real number ($x \neq 0$) and n is a positive integer, then:

$$\boldsymbol{a^{-n} = \frac{1}{a^n}}$$

To the **right** of the decimal point, the power that the base 10 is raised to corresponds to the name of the place value of the digit in the given quantity, as seen in the chart above (i.e. tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths).

To the **right** of the decimal point, the names of the place values repeat in the same pattern consecutively from left to right (i.e. to the right of millionths comes ten-millionths, hundred-millionths, billionths, etc.).

The **whole number part** of a given numerical quantity is represented to the **left** of the decimal point.

- When moving to the **left** of the decimal point, we are moving one place value **larger** with each consecutive column read **right to left of the decimal point**, as shown in the chart above.

It is important to tell students that when moving to the **left** of the decimal point, the given numerical quantity is to be **multiplied** by the corresponding multiple of 10.

To the **left** of the decimal point, the power that the base 10 is raised to corresponds to the name of the place value of the digit in the given quantity, as seen in the chart above (i.e. ones (or units), tens, hundreds, thousands, ten-thousands, hundred-thousands, millions).

To the **left** of the decimal point, the names of the place values repeat in the same pattern consecutively from right to left (i.e. to the left of millions comes ten-millions, hundred-millions, billions, etc.).

⚠ Students should be able to see that the names of the places to the **left** and **right** of the decimal point look similar.

- For instance, when moving one place to the **right** of the decimal point, the given numerical quantity in the **tenths** place.

- When moving two places to the **left** of the decimal point, the given numerical quantity is in the **tens** place.

Remind students that when moving to the **right** of the decimal point, a **-th** is added as a suffix to the name of the similarly named place value found on the **left** side of the decimal point.
The only exception to this is the **ones (or units)** place, which does not have a corresponding similar name on the left side of the decimal point.

- This is represented visually in the chart above. Each place value to the **right** side of the decimal point is a lighter shade of the same color as its corresponding similarly named place value found on the **left** side of the decimal point.

Using a Place Value Chart to Write Quantities in Expanded Form

Encourage students to complete the **place value chart** for the following values, and write the quantities in expanded form:

(a) 324
(b) 5.824
(c) 94.216037

millions	hundred-thousands	thousands	hundreds	tens	ones (or units)	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
			3	2	4	.	0					
					5	.	8	2	4			
				9	4	.	2	1	6	0	3	7

(a) *Row 1*:

- In the number 324:
- 3 is in the hundreds place.
- 3 means 300 because it is: 3 • 10^2

Recall: $10^2 = 100$

- 2 is in the tens place.
- 2 means 20 because it is: 2 • 10^1

Recall: $10^1 = 10$

- 4 is in the ones (the units) place.

  - 4 means 4 because it is: 4 • 10^0

🧠💭 Recall: $10^0 = 1$

💡**In expanded form, 324= $\mathbf{300} + \mathbf{20} + \mathbf{4}$**

⚠ Because we do not see the decimal point the whole number 324, it is located to the right of 4 (the last digit).

- We can add zeros to the right of the decimal point, but it is not necessary to do so.

(b) *Row 2*:

- In the number **5.824**:
  - 5 is in the ones (the units) place.
  - 5 means 5 because it is: 5 • 10^0

🧠💭 Recall: $10^0 = 1$

  - 8 is in the tenths place.
  - 8 means .8 because it is: 8 • 10^{-1}

🧠💭 Recall: $10^{-1} = \frac{1}{10} = .1$

  - 2 is in the hundredths place.
  - 2 means .02 because it is: 2 • 10^{-2}

🧠💭 Recall: $10^{-2} = \frac{1}{100} = .01$

  - 4 is in the thousandths place.
  - 4 means .004 because it is: 4 • 10^{-3}

🧠💭 Recall: $10^{-3} = \frac{1}{1{,}000} = .001$

💡**In expanded form, $\mathbf{5.824 = 5 + .8 + .02 + .004}$**

(c) *Row 3*:

- In the number **94.216037**:
  - 9 is in the tens place.
  - 9 means 90 because it is: 9 • 10^1

🧠💭 Recall: $10^1 = 10$

  - 4 is in the ones (the units) place.
  - 4 means 4 because it is: 4 • 10^0

🧠💭 Recall: $10^0 = 1$

  - 2 is in the tenths place

- 2 means .2 because it is: 2 • 10^{-1}

🧠 💭 Recall: $10^{-1} = \frac{1}{10} = .1$

- 1 is in the hundredths place
- 1 means .01 because it is: 1 • 10^{-2}

🧠 💭 Recall: $10^{-2} = \frac{1}{100} = .01$

- 6 is in the thousandths place
- 6 means .006 because it is: 6 • 10^{-3}

🧠 💭 Recall: $10^{-3} = \frac{1}{1{,}000} = .001$

- 0 is in the ten-thousandths place
- 0 means .0000 because it is: 0 • 10^{-4}

🧠 💭 Recall: $10^{-4} = \frac{1}{10{,}000} = .0001$

- 3 is in the hundred-thousandths place
- 3 means .00003 because it is: 3 • 10^{-5}

🧠 💭 Recall: $10^{-5} = \frac{1}{100{,}000} = .00001$

- 7 is in the millionths place
- 7 means .000007 because it is: 7 • 10^{-6}

🧠 💭 Recall: $10^{-6} = \frac{1}{1{,}000{,}000} = .000001$

💡**In expanded form, 94.216037 =**

90 + 4+.2+.01+.006+.0000+.00003 +.000007

Identifying the Number of Zeros in Large Numbers

⚠ Students often have trouble identifying **how many zeros are in large numbers**.

- The following chart, which corresponds to the **Place Value Chart** above, illustrates the number of zeros in the quantities:
  1 million, 10 million, 100 million, and 1 billion.

Name of Number	Number of Zeros in Number	Multiply 1 by:	Number
one million	6 zeros	10^6	1,000,000
ten million	7 zeros	10^7	10,000,000
hundred million	8 zeros	10^8	100,000,000
one billion	9 zeros	10^9	1,000,000,000